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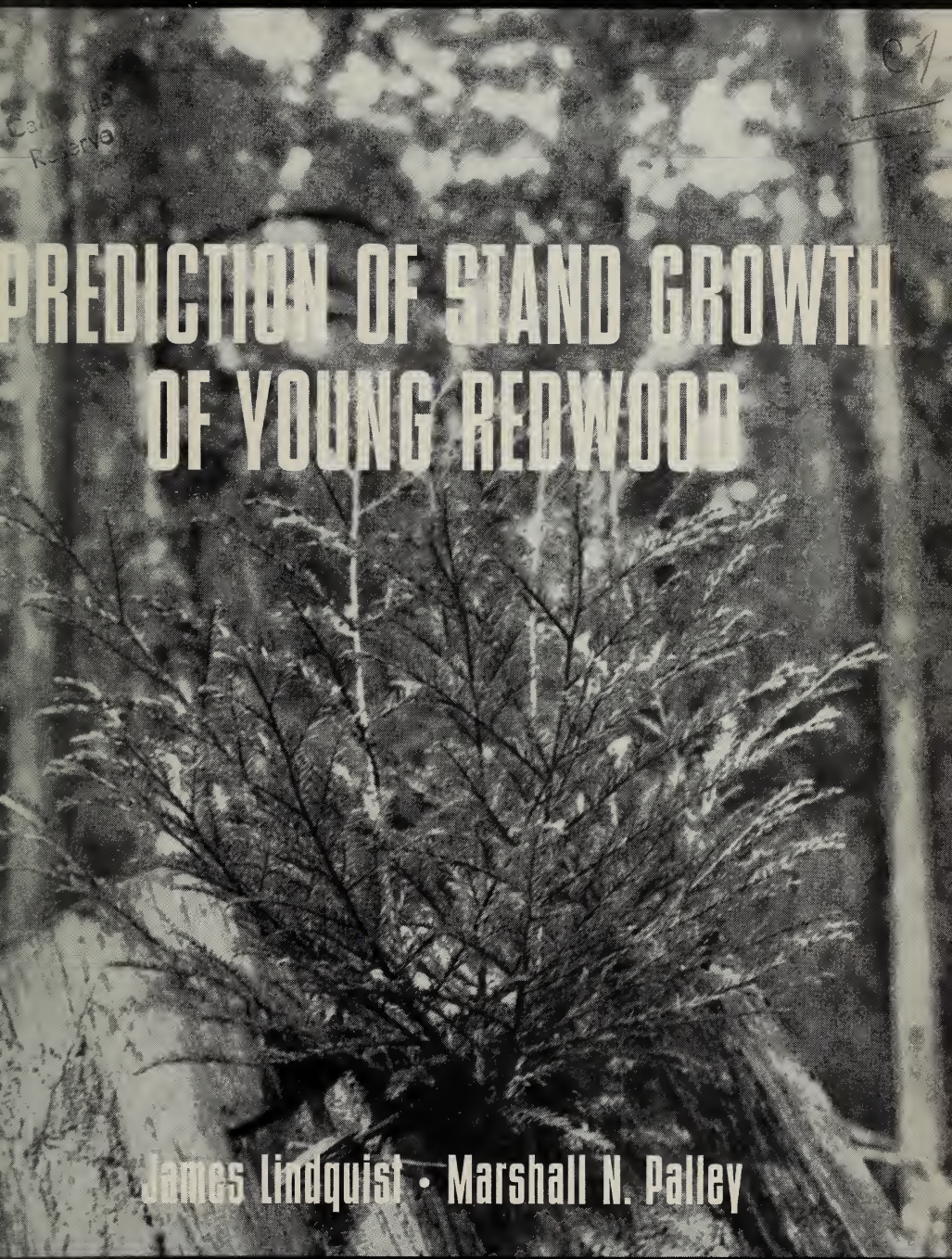
PREDICTION OF STAND GROWTH OF YOUNG REDWOOD



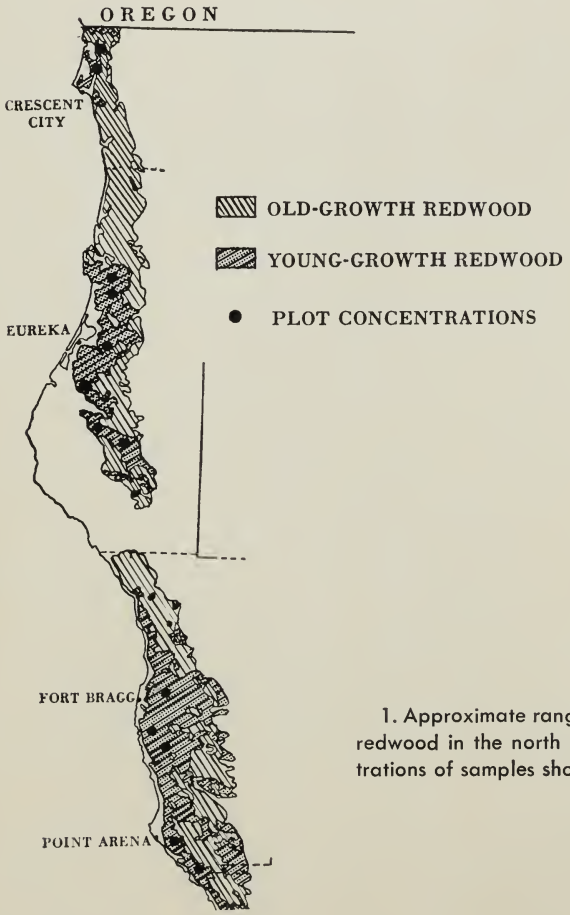
James Lindquist · Marshall N. Palley

CALIFORNIA AGRICULTURAL EXPERIMENT STATION

BULLETIN 831

This bulletin presents net ten-year periodic growth tables for even-aged young-growth redwood of the northern California coast counties to provide owners and managers easily usable predictions of their stand. It presents growth tables, prepared from multiple regression equations, for stands larger than 4.5 inches DBH and for stand segments larger than 10.5 and 15.5 inches DBH. To make use of the prediction process the grower needs to have current stand information of average breast-high age, site index (based on dominant redwood), and stand basal area of trees larger than 4.5 inches DBH. This information, secured from stand sample units, can then be used with the growth tables or equations to estimate future periodic stand increment.

This bulletin also contains information on site index curves, volume tables, sample frequency distributions, and developmental work carried out in the completion of this study. It aims at providing an integrated report of the developmental work and also a working document for those who need this information in field use.



1. Approximate range of young- and old-growth redwood in the north coast counties, with concentrations of samples shown.

The Redwood Region

Redwood grows in a narrow strip along the northern California coast, extending from southwestern Oregon to the vicinity of Monterey. This species seldom grows farther than thirty miles from the coast (figure 1). The major portion of this range with an economic potential for commercial timber operations is located in western Del Norte, Humboldt, Mendocino, and Sonoma counties. This bulletin is only concerned with young stands which developed since the mid-nineteenth century, after the virgin stands had begun to be harvested. The pattern of harvesting old-growth redwood has been strongly influenced by improvements of equipment and logging techniques for handling the massive logs of the old-growth stands. Cutting originated around bays and river mouths, then moved inland following rivers which provided routes for removing the logs; the level river flats which are the best sites were the first harvested. Acreage cut during this period of selective river logging was small, but now support the oldest stands available for study and represent the highest development of the young-growth type. Introduction of the train and powerful woods equipment, and later the more mobile truck and tractor had their impact on harvesting conditions under which these young stands became established. The majority of stands were established during the period of clear cutting, and this has resulted in large areas of relatively uniform age classes. Selective cutting in recent years, since the advent of the truck, has resulted in stands composed of mixed old- and young-growth trees.

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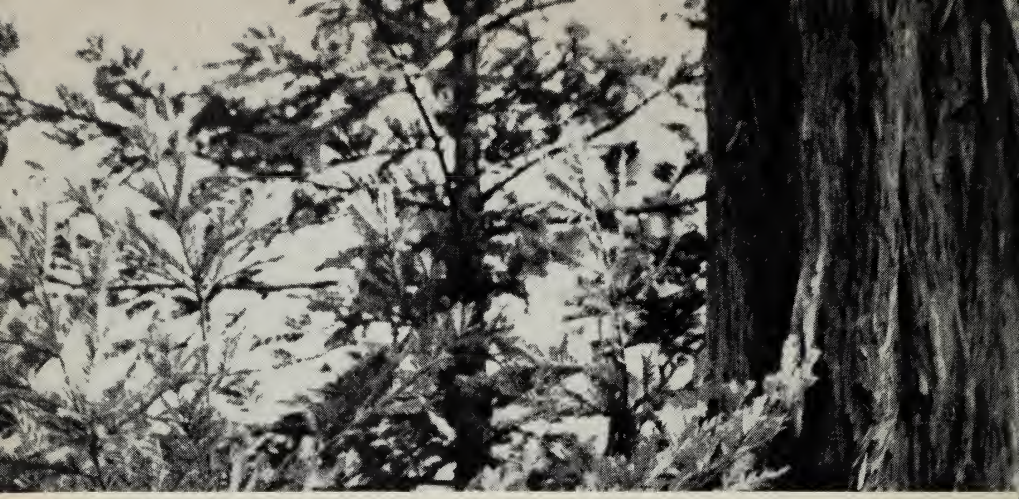


TABLE OF CONTENTS

	PAGE
Basis of study	6
Description of stands sampled	6
Sampling procedures	7
Reconstruction of past stand inventories	8
Calculation of data	11
Observations on the prediction process	12
Growth equations and tables	13
Basal area growth	14
Cubic-foot volume growth	17
Board-foot volume growth (trees larger than 10.5″ DBH)	22
Board-foot volume growth (trees larger than 15.5″ DBH)	28
Application of equations	32
Comparisons to other redwood growth studies	39
Summary and discussion	43
Bibliography	45
Appendices	
A. Stand inventory and growth computations	47
B. Sample distributions and descriptive information	52
C. Mortality considerations	55
D. Conversion of International volumes and growth to Spaulding log rule	56
E. Volume tables	58
F. Prediction error estimation	62

PREDICTION OF STAND GROWTH OF YOUNG REDWOOD[1, 2]

This study of young-growth redwood (*Sequoia sempervirens* [D. Don Endl.]) was designed for development of predictive equations of net ten-year growth of the stand characteristics; basal area, cubic- and board-foot volumes. Investigations of growth data, from 163 temporary point samples, have been carried out using multiple regression analysis.

Short-term growth predictions have a more direct application to actual stand management operations than do predictions from yield tables. This is particularly true when dealing with stands under consideration for harvest within a short period of time. The decision to cut a stand now or leave it for a period of time may often be based on the expected growth over the next few years. Stand growth is a complex system involving soil, climate, species of trees comprising the stand, and the physical occupancy of the land by the trees. Within a stand the gross growth added to the existing stand structure over a period of time is the accumulation of growth on each individual surviving tree. This is an oversimplification when we consider that stand growth also is affected by loss and ingrowth of entire stems. Overall effects of forces existing within a stand structure cannot be fully understood or explained by reactions (growth) to a single influence. Commonly the summation of environmental influencing factors is included in a single expression, site index. This index of land productivity measures height growth of dominant trees of a given species for existing local environmental conditions. Site index, in this context, permits a rating of land productivity without need of fully understanding underlying causes of the qualitative reaction. Despite combining climatic and soil-factors influences on a tree species height-growth pattern into a single term, the problem of assessing growth remains complex because of growth reactions to stand factors such as age, stocking, and stand structure. Recognizing the improbability that any measure of stand growth is a function of a single site or stand characteristic has caused foresters to resort to many devices for estimating future stands. Some of these include the stand table projection, the normal yield table, and two-way method of prediction, to name but a few.

Prediction of the unknown magnitude of a variable may be accomplished from established relationships between observations of this variable and measurements of other variables associated with it: a regression estimate. This, in its simplest form, is the linear regression of "y" on "x." This simple linear equation is unlikely to satisfy the complexity inherent in forest stand growth. Investigations must consider two or more independent variables.

Measurements of five- and ten-year net growth of several stand characteristics and the initial inventories upon which this growth is made have been made on 163 temporary point samples. Multiple regression analysis of the

[1] Submitted for publication March 2, 1966.

[2] The study reported in this bulletin was supported in part by funds provided under contract from the California Division of Forestry.

stand data has related growth to some characteristics of the initial stand and calculated regression coefficients that express this relationship. The calculated regression coefficients, from this sample, are to be used to predict ten-year net stand growth of stands for which only the initial inventory characteristics are available.

BASIS FOR STUDY

Source of growth information used to prepare these multiple regression growth equations and associated growth tables was 163 temporary point-samples of young-growth redwood. These sample units, established in 1958 and 1959, represent a cross-section of the even-aged unmanaged relatively pure stands of this species growing in the four north coast counties of California. Derivation of stand basal area and volume growth was done through reconstruction of the initial stand inventory of each sample unit from five- and ten-year radial increment cores extracted from each living tree. Aspects of the study discussed in this section include: a brief description of the stands sampled, establishment and measurement of the samples, reconstruction of past inventories, data calculation, and the general approach to the derivation of the prediction equations.

Description of stands sampled

Stand sampling for this study of young-growth redwood was carried out in the four counties mentioned; concentrations of samples are shown in figure 1. The sampling was to measure growth of representative stands reflecting the variety of age, site, basal area, and topographic conditions existing in the geographic localities examined. Only relatively pure and undisturbed even-aged stands of young-growth redwood were considered. These criteria resulted in exclusion of stands which showed evidence of recent disturbance either from cutting or extensive blowdown. During harvest of the original stands many poor-quality or small trees were left standing; these trees remain as an integral part of the present young stand. Where this occurred, samples were located so as not to include either residuals or young trees that were strongly influenced by the residuals. Young stands that originated under partial canopy of old-growth trees were not included in this sampling, because this in effect gives a two-storied stand not compatible with the even-aged criteria.

Degree of stocking of the stands was not strongly considered in the sampling. The question of a "normal" level of stocking was not a basic criterion when choosing stands included in the representative sample. More specifically, stands were included if sufficiently stocked to produce a manageable and economically valuable crop. For the stands sampled, average initial stand basal area per acre, an index of stand density, of all trees larger than 4.5 inches in diameter at breast-height is 306.7 square feet with a standard deviation of ± 190.7 square feet. A comparable value for the terminal inventory is 401.4 ± 194.8 square feet. Mean site index of dominant redwood is 175 ± 23.1, and average initial stand age is 40.4 ± 22.9 years at breast-height. Species composition of the initial stands indicate that the percentage of redwood basal area

averaged 86.5 per cent; other conifers, 11.5 per cent; and hardwoods, 2.0 per cent; approximately 90 per cent of the stands had in excess of 90 per cent redwood of the total basal area. It should be noted that point sampling selects trees on the basis of diameter and distance from the sample point; thus many stand samples which indicate 100 per cent redwood may have included trees of other species, but not of a proper size to be selected by the sampling system. Distribution of samples by stand and site conditions, shown in tables 16, 17, and 18, provides further indication of stand conditions encountered in this sampling. Basing prediction equations on initial stands made it necessary to show sample distributions in terms of the reconstructed initial stands. Site index is based upon the average age and total height of dominant redwood as measured; site index is assumed to have remained constant during the growth period. Origin of the stems, whether sprout or seedling, indicated that the majority of the redwood stems established in these stands were stump sprouts. Approximately 35 per cent of the samples have entirely sprout-originated stems, 65 per cent of the redwood stems were considered as sprouts.

Sampling procedures

The sampling unit in each stand was a cluster of trees selected through use of an angle-gauge. The relascope provides a choice of basal area factors and automatic adjustment for sloping ground. The point-sampling technique (Grosenbaugh, 1952) appears ideally suited for growth studies using temporary samples which require extraction of increment cores. A major advantage of this sampling technique is that fewer trees are required to estimate the stand values. By selection of the trees with a probability proportional to their basal area, the valuable large trees are more intensely sampled. Thus growth as measured on larger trees receives a proportionally heavier weight than that of the smaller diameters in the samples. Depending on the stand conditions, the choice of the basal area factor (BAF) was made to insure that a sample of 15 to 25 trees was measured. The basal area factors employed were: 10, 20, and 40.

Selection of the stands for sampling was made to complete a cross section of ages, sites, and stocking in each of the localities visited. Thus with purposeful rather than random sampling, stands were selected to fill a gap in the data. Positioning the sampling point within a stand was accomplished by making several preliminary stem counts to determine the approximate stand basal area. The point established represented the average of these basal area estimates of the stand conditions. A mounted staff compass was set at this point and each tree checked with the angle gauge. If the diameter image at breast-height exceeded the width of the selected basal area factor, the tree was included in the sample. When the entire diameter of the tree was not visible, or if the tree was near the limit of acceptability, the horizontal distance from the point to the midpoint of the tree was measured. This point-to-tree distance, if less than the product of the diameter (inches) times the plot radius factor (a constant dependent on the basal area factor) meant that the tree was to be included in the stand sample.

All living trees, and those estimated to be dead 10 years or less, whose diameter at breast-height exceeded 4.5 inches were eligible for the sample.

Beginning at 0° azimuth, each tree as selected was plotted on a stem map and recorded on a plot recording form. Information recorded for each tree included: diameter at breast-height; species; crown class; origin, seedling or sprout; and five- and ten-year radial growth. Diameter measurements (.1 inch) were made with a diameter tape at 4.5 feet above ground on the uphill side. Breast-high radial growth was measured to the nearest .01 inch from increment cores removed from each living tree. Cores were extracted from the tree side facing the sampling point. Measurement of increment was made from the outside edge of the summer wood of the last complete growth ring, 1957, to the summer wood of 1952 and 1947. Sampling during the summer of 1959 measured the same set of growth rings despite the fact that growth of the 1958 season was complete; this established a consistent period of years over which radial increment is recorded.

Total height and breast-high age were determined on three to five dominant redwood in each sample. These measurements were made to determine average stand age and total height, the basis for site index evaluation. In addition, heights and diameters of these trees function as the basis for calculation of stand volumes, as explained more fully later in this bulletin. Samples which included other conifers required at least one measurement of height for volume calculations; where possible a dominant tree was used. Often the only tree, within the sample, of these other conifers was of an inferior crown class.

Cause of death and length of time since death were estimated from physical evidence. The causative agent was not critical to subsequent computations; however, time elapsed since death dictated the last inventory to which that tree contributed living volume. Two distinctions were made for included dead trees: those dead zero to five years, and those dead five to ten years. Estimates of the time of death are relevant to the determination of the periodic mortality, and also to reconstruction of the initial inventories.

Reconstruction of past stand inventories

Temporary samples were necessary for this study because of the length of time required for permanent growth plots to provide required growth measurements —a minimum of ten years. A few permanent growth plots exist in this timber type, but not in sufficient numbers to provide the broad data base required in projects of this type. Furthermore, the growth plots available were established for a variety of reasons, resulting in a lack of continuity in measurements recorded. The most valuable aspect of these permanent plots is that they can and will function as a check against the results of predictions made from the data collected in this study. Temporary samples require measurements of recent periodic diameter and height growth to reconstruct a stand inventory ten years prior to sample establishment. Several problems peculiar to reconstruction of past stem size need to be resolved, because a recovered past inventory provides the stand growth and growth base from which to develop the prediction equations. Success of prediction rests initially on accurate tree measurements of periodic radial and total height growth of the individual trees of a sample. Removing growth cores and determining five- or ten-year radial increment was not ex-

cessively time consuming using point-sampling, because samples seldom had more than 25 trees. The reconstructed initial stem diameter outside bark could not be determined by subtracting radial growth from the present outside bark diameter, because during the growth period the bark also increased radially. The effect of this bark growth during the ten-year period cannot be ignored because diameter measurements outside bark refer to the woody volume inside bark, a feature of the volume tables used in this work. The radial wood growth was adjusted to reconstruct initial outside bark diameter.

Bark thickness is related to stem diameter, not age (Bruce and Schumacher, 1950). Increment of bark over a period of years is a function of the initial stem diameter and the periodic increment of woody tissue of each stem. Establishing a relationship between diameters inside and outside bark at breast height permits evaluation of the growth of bark per unit of wood growth. Diameter outside bark plotted over diameter inside bark normally indicates a linear relationship between these two stem dimensions (Chapman and Demeritt, 1936), with the double-bark thickness for a given diameter as the difference between this line and a 45-degree line. Knowing present diameter outside bark and radial periodic growth of wood, makes it possible to determine the past inside and outside bark diameters. A diameter outside bark (DOB) over diameter inside bark (DIB) curve was established from 50 young-growth redwood trees representing the range of stem diameters found in the sampling for this project. This plotted data indicated a straight line that intercepted the DOB axis at approximately the origin (zero). Removal of the intercept value was done by calculating the regression with the restraint that the line be forced through zero. The calculated regression coefficient, 1.1598, agrees closely with that found by the graphical fitted line:

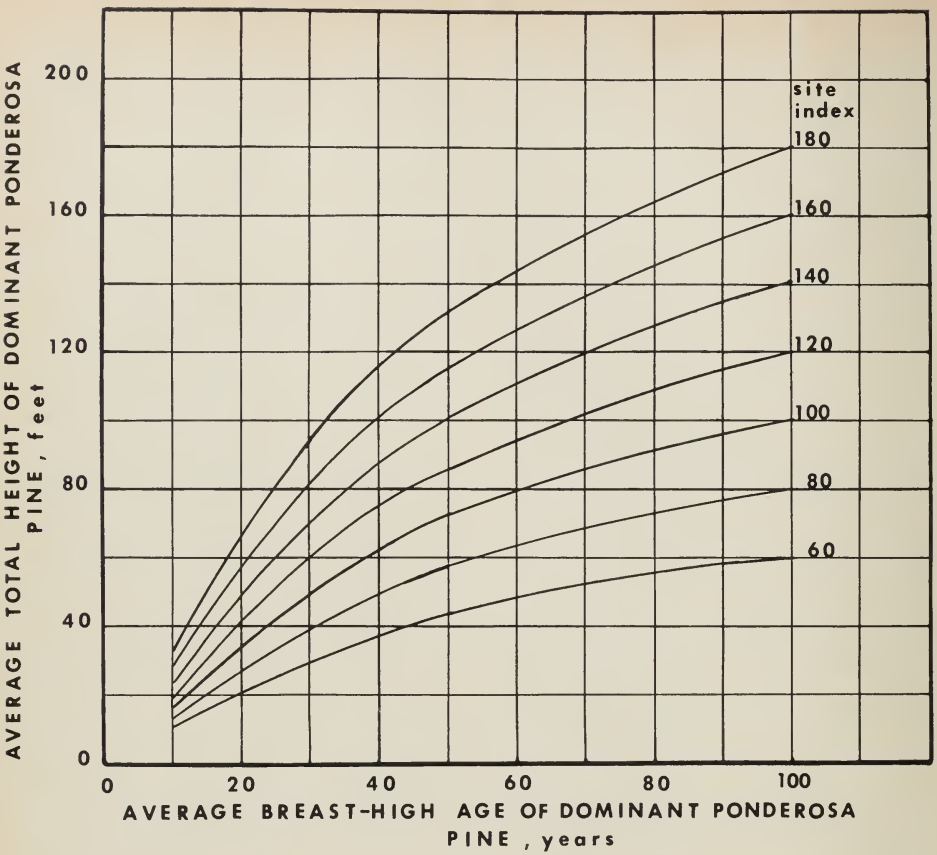
$$\text{DOB} = 1.1598\ (\text{DIB})$$

This relationship expresses zero DOB at zero DIB and states the rate of increase of diameter outside bark per unit of increase inside bark. Young-growth redwood diameter outside bark increases approximately 1.16 inches per inch of inside bark diameter increment. Calculation of the initial inventory diameter outside bark of each stem in a sample was done by using the following expression:

$$\text{Past DOB} = \text{present DOB} - 2(1.16 \times \text{radial growth})$$

Values of bark growth determined in this study are comparable to those of Pemberton (1924), who found bark thickness to be approximately 15 per cent of outside bark diameter at breast height for young-growth redwood trees. Value of bark growth for recovery of the past diameters of trees other than redwood, is 1.13 inches of outside bark growth per inch of radial wood growth.

Site index as an independent variable in this study is based on site index curves developed from this basic sample data (Lindquist and Palley, 1961). Growth and stand inventory values used in these predictions are keyed to these site curves, in terms of the breast-high age and total height of dominant redwood, as shown in table 1, and figure 2. Redwood does not have a recognizable annual internodal height growth pattern. This creates a problem of determining the initial total height of those trees whose terminal height was



2. Site index curves showing average height of dominant redwood by site index and breast-high age class.

Table 1. Average Total Heights of Dominant Redwood by Breast-High Age and Site Index

Age at b. h.	Total height, by site index							
	100	120	140	160	180	200	220	240
years	*feet*							
20.....	21	31	42	53	63	74	84	95
30.....	34	48	62	76	89	103	117	131
40.....	47	62	76	92	107	122	137	152
50.....	57	73	89	106	122	138	155	171
60.....	67	84	102	119	136	154	171	188
70.....	76	94	112	130	148	166	184	202
80.....	85	104	123	142	161	180	198	217
90.....	93	113	132	152	171	190	210	229
100.....	100	120	140	160	180	200	220	240

measured. Initial total heights of these trees are required to calculate stem volumes necessary for the determination of the past inventory local volume tables. Solution of this problem was done by assuming that patterns of tree height growth shown by the site curve approximate the rate of stem elongation. The measured terminal breast-high age and total height of each tree is an entry point from which to follow the site curve of height back ten years to estimate past stem total height. This procedure assumes that the site index does not change over the ten-year

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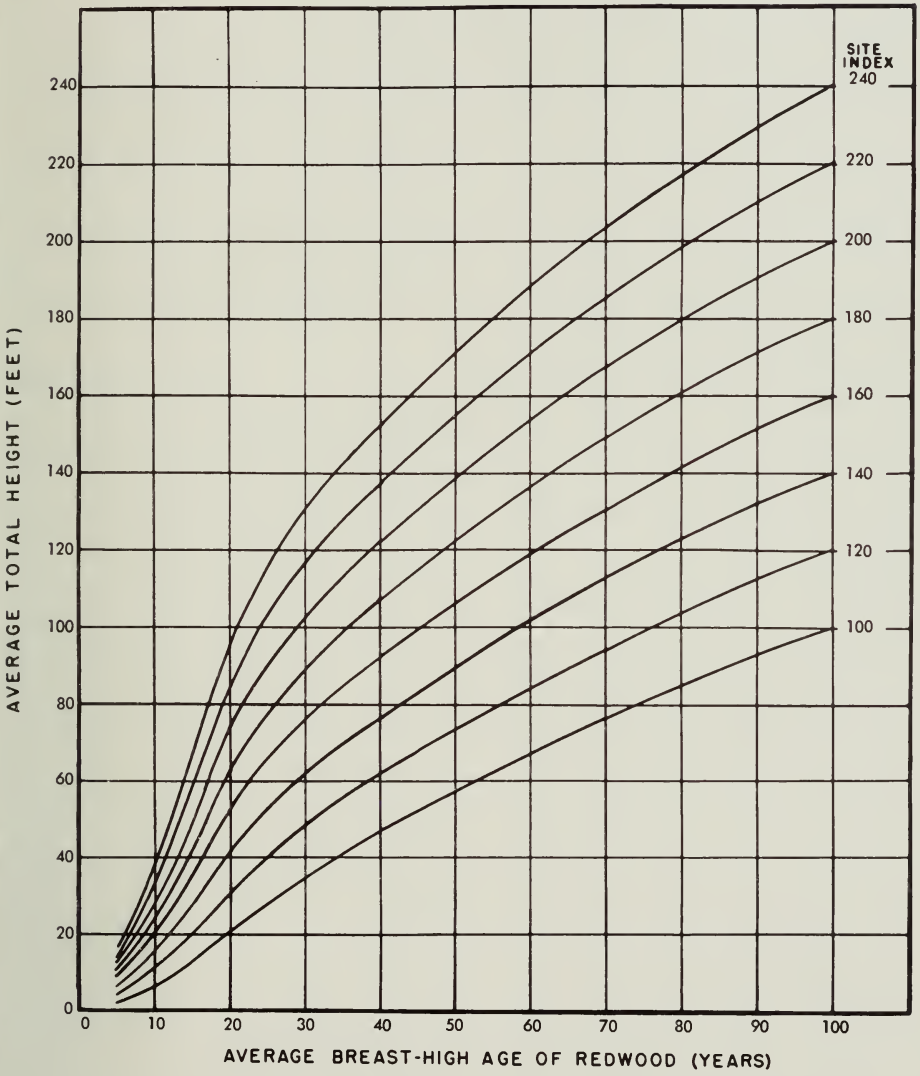


On pages 25 and 26, in table 10, and on pages 30 and 31, in table 13, the Volume Growth figures are in board feet, not square feet.

period, and also that the stems have maintained a dominant position in the crown over the growth period. There is, however, no assurance that the height growth of a particular tree will follow this pattern, but this assumption is required because no other method of estimating stem growth is available when temporary plot measurements are used.

Use of site index as rating of the land productivity is done by incorporating breast-height age and total height into a single term, the total height dominant trees should attain at 100 years, the base age. Use of site index in the prediction equations is made by treating site index as an independent variable which functions as the rating of relative growth potential of land.

Calculation of data

Reconstruction of initial stem diameters and total heights has prepared the way for calculation of initial and terminal stand inventories, and the resultant net periodic growth. Use of point samples for stand inventory evaluation requires conversion of sample tree measurements to per-acre values with formulas designed to express the contribution of each tree in the inventories at a rate inversely proportional to its measured basal area (Grosenbaugh, 1955). Sample trees receive the same weight, based on the terminal basal area, in each of the inventories considered. This weight in the terminal inventory is related to its probability of selection in the sample, a function of its measured basal area. For the initial inventories, a tree's contribution to stand basal area is expressed by the ratio of its past basal area to its present basal area; if the tree has grown during the period, this value is less than one.

A computer program was designed for developing the required inventory statistics descriptive of the sampled stands. This program processed data through a sequence of computational operations to convert tree measurements to values of stand frequency, basal area, cubic-foot and board-foot volumes, and average diameter. Basically, the program consists of two main segments: classification operations, and the per-acre calculations. A more descriptive summary of calculating operations is contained in appendix A.

A brief summary is provided here of the type of information derived for each of the 163 samples. The classification portion of the program evaluated each tree in light of several restrictions to determine categories into which the tree would contribute its values. Three distinct species classes were used: redwood, whitewood (all conifers other than redwood), and hardwood. A second series of categories was related to the three inventories to be calculated; these are: the present inventory, based on actual diameter and height measurements, and past inventories, based on five- and ten-year radial and height growth (plus allowances for periodic mortality and ingrowth). For each species an inventory calculation of three minimum diameter stand segments was done; trees larger than 4.5 inches diameter at breast height were included in the cubic-foot stands, and trees larger than 10.5 and 15.5 inches diameter for computations of the board-foot values. Further identification was made of trees which exceeded the minimum diameter limits during the growth period, ingrowth; and trees that died during the period, mortality. The function of the classification portion was to examine each tree and place its

contribution, based on species and present or reconstructed diameter, into the proper categories. Values accumulated in each category were the reciprocal of the square of the terminal diameter, and the ratio of past to present diameter. The calculation portion of the program converted the data accumulated in the classification section into per-acre values for each category, using a series of appropriate formulas. Total values of each characteristic in each inventory and diameter limit were found by addition across the three species groups. These total inventory values and the growth for the five- and ten-year period were then printed in tabular form.

Development of stand cubic- and board-foot volumes from the sample tree measurements is predicated on calculation of a series of local volume lines for each of the coniferous species in each inventory. Derivation of the local volume lines depended on calculation of stem volumes for trees whose total height was measured. Stem volumes of these trees were evaluated from standard volume tables expressed as formulas (table 24) using the appropriate inventory diameters and heights. The computed local volume lines were expressed as simple linear regressions of stem volume over diameter squared. Stand volume calculations used these local volume line regression coefficients in conjunction with accumulations of the classification section to derive estimates of stand volumes for each category.

Observations of the prediction process

Multiple regression equations are calculated which show relationships between stand growth and related initial stand and site values. These coefficients provide a means of estimating the relationships between values of stand responses to initial stand and site influences. Investigations of forest stand growth within a complex mathematical system through use of multiple regression analysis permits testing of a number of independent variables in an exploratory manner. The problem is to quantify the relationships of the various measures of stand growth to reconstructed independent variables of the inventory date preceding the growth period. Examining stand growth reactions within this system resolves to selecting the most reasonable fit of the sample data relative to: (1) precision of the estimate, (2) type and number of measured stand independent variables, and (3) a configuration consistent with plotted growth trends. Preciseness of estimate is best expressed by the standard error of estimate, SE_{est}; this value expresses the standard deviation of differences between measured and regression equation estimated growth. This statistic is most valuable as our interest lies in making the best possible predictions of stand growth; i.e., the smaller the standard error the better the estimate agrees with the measured growth data. The coefficient of multiple determination, R^2, indicates amount of total variation present in the dependent variable explained by the regression equation. Practicality of the selected equation needs to be considered because the forest owner has a useful tool only when the prediction system is amenable to convenient use. Three types of independent variables are defined to clarify questions that may arise over repeated use of this term. We consider measurements of stand characteristics such as age, basal

area, site, height, and volume as basic stand variables; beyond this are transformations of these variables such as the logarithm, reciprocal, square, and so forth. Combining two or more basic variables or their transformations into a single variable is also used. Using basic, transformed, and combined forms of the stand and site data develops for study a large number of independent terms from which to derive useful prediction equations.

Inclusion of more than three basic variables as independent predicting terms creates problems when using either graphical or tabular representation of stand growth; they can only be feasibly used in an equation form. Only if an equation based on four or more *basic* variables is so clearly superior to those of three or less would there be a justification for recommending it for practical application. Use of stand or site independent variables which are difficult to measure, even if proving to be useful in the prediction of growth, are not included in the equations. These aspects of practicality for field use were considered when deciding to accept a growth prediction equation. Under certain circumstances an equation may be easily applied and precise, yet unacceptable because the position or configuration of the growth curves is unrealistic. Through solution of equations and plotting results it is possible to understand the combined effect a set of independent variables has on stand growth.

Growth curves, derived in this manner, show expected reactions of stands with magnitude of the independent variables fixed; they are for static conditions at a fixed point in time. All stand variables, except for site index, are in a continual process of change; as result of tree growth the prediction base is constantly in flux, reflecting different rates of periodic growth.

Growth sampling in the manner outlined has attempted to provide a reasonably accurate picture of stand growth as it occurred in the broad range of conditions existing in these stands. Errors in selection and measurement of sample stands, site curves, derivation of past stem diameters, volume tables, and other features incorporated in this study all have a bearing on the final outcome of the predictors. These errors plus those associated with the errors always present when attempting to estimate population parameters through sampling should be recognized by users. In subsequent application of these equations errors associated with sampling and measurement will be in operation. By careful and proper use of the prediction equations the users will not compound the problems inherent in a system of growth prediction derived from an empirical sample.

GROWTH EQUATIONS AND TABLES

Preparation of the stand net-growth tables described in this section followed a generally similar pattern. The first phase of the investigations was exploratory, looking at simple regressions and correlations of stand growth on a variety of independent variables (table 23). These simple correlations served to point directions that might prove profitable in subsequent multiple regression analysis.

The initial step of the multiple regres-

sion analysis considered the untransformed dependent and independent variables in linear multiple regression equations. From this step the examination of data moved into consideration of the variable quadratic and cubic forms. The mathematical models, linear and quadratic, were also tested when substituting the logarithm (base 10) of both dependent and independent variables. These tests provide a degree of analysis which is compatible with the formal approach to data analysis when little information about the form of the relationships is known. Additional investigations involved using transformations and combinations of independent terms not considered in the formalized models. As a result of these testing procedures, within the multiple-regression framework, a large number of equations were considered for each of the stand growth factors studied. The general consensus that evolved from the tests was that the three basic variables—stand age, stand basal area, and site index—would effectively serve as the basis for the predictions of net stand growth. Equations have been selected on the basis of the improvement or reduction of the standard error of estimate, and also on the configuration of resulting growth curves. The SE_{est} of the untransformed dependent variable is compared with that of the logarithmic transformation by use of the index of comparison reported by Furnival (1963).[3]

Basal area growth

Stand basal area per acre is a measure of stand density expressing the degree of land occupation by trees (Bickford, 1953). Easy to measure, this stand value functions as a single term indicative of the growing base of a stand. Basal area alone is not entirely descriptive of stand structure for it does not provide a complete view of the stand; nor does basal area alone provide a full indication of its effect on the yield or growth of a given volume unit. This latter aspect is seen by the wide range of volume yields and growth on stands of equal basal area. This implies that a meaningful investigation of stand growth should include other stand and site characteristics to provide the qualifying effects on stand basal area.

Expressing stand growth as an increment of stand stem area provides an index of growth capacity relatively unaffected by: (1) site index, as measured by height and age; and (2) volume units, which are affected by restrictions of stem segments and tree size. The value of a study of basal area growth is its function as an indicator of stand response to environment specifically related to the trees of a stand. Basal area growth shows more clearly than does a volume unit (based on stem height and basal area) the influence of individual trees which comprise the stand. This effect becomes clearer when considering the reaction of an individual tree to its immediate neighbors.

Growth predictions of stand basal area are based on the initial inventory stand of all trees larger than 4.5 inches at breast height, and refer to the increment of this stand. Average ten-year periodic growth of 163 stand samples is 94.6 $\pm$ 46.6 square feet. Comparable values for the stand larger than 10.5 inches (162 samples) are 94.5 $\pm$ 46.6 square feet; for the stand larger than 15.5 inches (158 samples) 93.4 $\pm$ 46.3 square feet. Generally, basal area

[3] Index of comparison SE_{est} normal units = $SE_{est\ log} \cdot$ Antilog $\bar{Y}_{log} \cdot 2.3026$.

growth has low correlations with the independent variables considered in these investigations (table 23). The regression of basal area growth on stand age indicates a negative slope, growth decreases with increasing age. During the young ages, trees in a stand have sufficient room to expand and utilize the maximum potential of the site. Consequently, increment can be maintained at a reasonably high level. As age increases, trees become more competitive for the existing space and nutrients; the increasing diameter, crown and roots bring trees into closer contact. Intense competition which may develop within a stand as trees grow results in differentiation of stems into crown classes, death of trees that have not gained a favorable position in the developing canopy, and reduction in radial increment.

The linear three variable model (table 2) is the first step in analysis of multiple variable models tested in this study of basal area growth. In this linear form, site index is the least valuable in explaining the variation of stand basal area growth in this sample (t-value).

In developing this equation via a stepwise procedure it is noted that the SE_{est} reduces from ±38.41 sq ft, with only age and basal area to ±38.09 sq ft with introduction of site index. Initial stand basal area, when used in conjunction with stand age, becomes more important than previously indicated by the simple correlation coefficient of basal area growth and initial stand basal area (.033). The predictive ability of these three variables is improved when the quadratic and cubic forms of the three independent variables are used; the coefficient of multiple determination and standard error of the estimate are respectively: quadratic, .702 and ±33.81 sq ft; cubic, .573, and ±31.22 sq ft. In these models interaction terms that include site index take on higher t-values and importance in the explanation of growth variability. However, in plotting the quadratic and cubic equations with fixed age, site, and basal area levels it becomes apparent that the growth patterns are diverse and a clear picture of the effects of these variables is not developed. This and the fact that relatively similar standard errors and coefficients of multiple determination are derived from less complex equations, led to further testing basal area growth equations using logarithmic transformations of the independent and dependent variables.

Table 2. Linear Equation of Basal Area Growth on Age, Site Index and Basal Area for All Trees Larger than 4.5 Inches DBH

Variables	Reg. coef.	Std. error	t	Beta coef.
Age	−1.63927	.21008	−7.80	−.809
Site	.28531	.14852	1.91	.141
Basal area	.12729	.02794	4.63	.52

Constant = 71.73
Std. err. of est. = 38.098 sq ft
Mean BA.Gr. = 94.60 sq ft
Coef. var. = 40.27 per cent
Coef. of mul. det. = .3437

Subsequent to transformation of the variables into logarithmic form (base 10) testing was carried out with the linear and quadratic models. The linear three variable model has a logarithmic standard error of .1403, which when converted by Furnival's index is ±27.54 sq ft, and a R^2 of .533. Similar values for the quadratic model are .14005 equal to 27.41 sq ft, and R^2 = .541. In this linear model the logarithm of site index has a negative coefficient, indicating a decreasing basal area

Table 3. Prediction Equation for Values of Net 10-Year Basal Area Growth of Stand Larger than 4.5 Inches DBH Shown in Table 4

Variables	Reg. coef.	Std. error	*t*	Beta coef.
Log age.....	−.975706	.07313	−13.34	−1.135
Log BA.....	.587816	.0543	10.82	.921

Constant = 2.03040
Std. err. of est. = .14071 (× 84.875 × 2.306 = 27.531)
Mean BA.Gr. = 1.92878 (Anti Log = 84.875)
Coef. of var. = 32.44 per cent
Coef. of mul. det. = .527

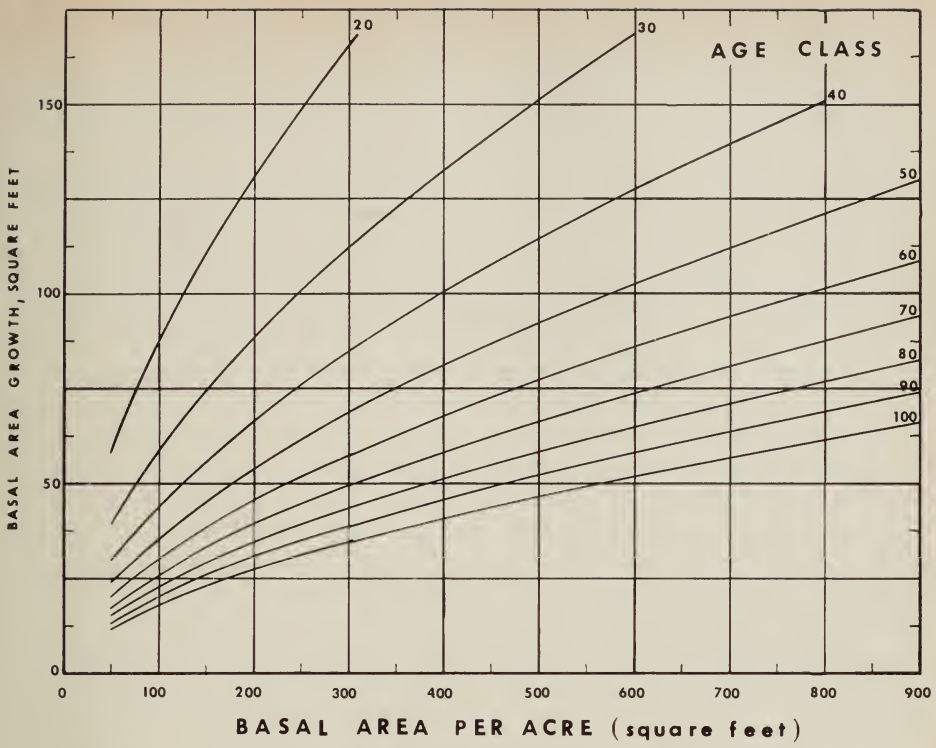
growth as site index increases. This apparently indecisive feature of site index in the equations tested, plus the fact that site index alone is generally not significant to the reduction of growth variation, has resulted in the selection of a simpler model to predict future ten-year basal area growth. The equation using the logarithmic transformations of basal area growth, initial stand basal area, and stand age, shown in table 3, has been used to prepare the basal area growth values in table 4 and figure 3.

This version of the stand basal area growth situation disregards effects of all other growth influencing variables except initial stand age and basal area. The absence of site index term in this equation is a result of the aberrant nature of the site index term in the many equations tested, specifically in this linear logarithmic model, a negative effect of increasing site index. Other testing of the data which included the independent variable number of trees per acre indicated that little improvement was made in prediction of future basal area growth over that of the model selected. The nature of the variation in stand basal area growth of this sample is such that regardless of the complexity of the equations investigated the standard error of the estimate did not reduce to less than 30 per cent of the mean basal area growth. Nor was more than 60 per cent of the total variation explained by an equation.

Reduction in basal area growth during the first 50 years is rapid, indicating that those stands which do not have sufficient regeneration may not develop a basal area level commensurate with the potential of this species. It is also evident that once the stands have reached 50 years the differences in

Table 4. Net 10-Year Basal Area Growth of Stands, Trees of All Species Larger than 4.5 Inches at Breast height

Age	Basal area per acre (sq ft)									
	50	100	200	300	400	500	600	700	800	900
	Basal area growth (sq ft)									
years										
10.....	113.8	171.1	257.2							
20.....	58.0	87.2	131.1	166.3	196.9					
30.....	39.1	58.8	88.3	112.1	132.8	151.4	168.5	184.5	199.5	
40.....	29.5	44.4	66.8	84.7	100.4	114.4	127.4	139.5	150.8	
50.....	23.8	35.7	53.7	68.2	80.8	92.1	102.5	112.2	121.4	130.1
60.....	19.9	29.9	45.0	57.1	67.7	77.1	85.9	94.0	101.7	108.9
70.....	17.2	25.8	38.7	49.2	58.2	66.4	73.9	80.9	87.5	93.8
80.....	15.1	22.6	34.0	43.2	51.1	58.3	64.9	71.1	76.9	82.4
90.....	13.4	20.2	30.3	38.5	45.6	52.0	57.9	63.4	68.6	73.5
100.....	12.1	18.2	27.4	34.8	41.2	46.9	52.2	57.2	61.9	66.3



3. Net ten-year basal area growth per acre for all trees larger than 4.5 inches DBH, for fixed initial breast-high age and basal area classes.

growth rate of the various levels of initial basal area become increasingly smaller as the stand age increases. The lower basal area levels are not as adversely affected by increasing age as are the higher classes in terms of absolute square feet of basal area. Considering the high degree of correlation of stand volume, regardless of volume unit, with stand basal area, one must see that measures to insure good initial stocking at stand regeneration time are of vital importance in securing a good future basal area level on which the volume yields are developed. These curves should not be interpreted to mean that site does not affect the level of stand basal area at a given age. Rather they imply that at a given age and basal area level the rate of basal area growth should be the same regardless of the site index.

Cubic-foot volume growth

Cubic-foot volume expresses more accurately actual wood present in a tree or stand than does board-foot volume. As Spurr (1952) points out, board-foot units are arbitrary and predicated on use of the wood. Cubic-foot content includes wood not entirely available as an economic product because of losses incurred through sawing, stem taper, and minimum dimensions of trees which can be utilized. Despite utility of board-foot units in commerical transactions, assessment of volume growth rates and yield of land is more accurately evalu-

ated through a study of cubic-foot content. Volume of a tree is commonly regarded as a function of its height, basal area, and form. Since trees show a degree of taper along the stem, it is necessary to consider how this taper affects stem volumes. Problems regarding types of geometrical solids that stem segments approximate are dealt with in the construction of volume tables for a given species. Recognition that stem form is considered in the construction of volume tables allows this study to proceed without further consideration of this matter. Stand cubic-foot volume is expressed as a function of stand basal area and total height; stand volume growth is affected and measured by changes over time in these two stand measurements. Stand cubic-foot volume growth refers to growth of all trees of all species more than 4.5 inches in diameter at breast height. No deduction for cull, breakage, or defect of the stems is assumed; stand growth refers to the net ten-year periodic increment of the stands.

Volume growth is more closely allied to site index than is basal area growth, since volume increment depends in part on vertical growth of the trees. In this respect, volume is sensitive to environmental effects of soil and climate. Whether using site index or values of height and age, it is important to consider the periodic vertical extension of stands when studying a measure of volume growth. Site index provides a method of grouping land areas of equal height-growth potential regardless of age of stand currently occupying the land. Preliminary information regarding the relationship of site and volume growth of these 163 sampling units is determined from the simple correlation coefficient .624. This states that the relationship is one of increasing volume growth with site, and that relative to other independent variables considered site is a good indicator of cubic-foot growth.

The second element affecting stand volume, stand basal area, represents the horizontal dimension of the stand. The simple correlation coefficient of volume growth and initial stand basal area, .513, is indicative of a limited degree of relationship. Plotting volume growth over the initial stand basal area shows a curvilinear pattern, with growth reaching a maximum at approximately 700 square feet. Initial stand basal area with age (simple correlation, .755) means that the higher initial basal area levels will normally be associated with the older ages.

Stand cubic-foot volume growth shows a low degree of correlation with initial stand age, .056, virtually unrelated. The importance of age is more fully realized when combined with other independent variables. Basal area divided by age has the highest simple correlation of any of the independent variables tested in this investigation, .835. This is indicative of a high linear relation of volume growth with this ratio of two stand variables.

Consideration of prediction equations began with age, site index, and basal area in their untransformed

Table 5. Linear Equation of Net 10-Year Cubic Foot Volume Growth

Variables	Reg. coef.	Std. error	*t*	Beta coef.
Age	−50.9738	6.422	−7.937	−0.624
Site	28.7389	4.571	6.287	0.353
Basal area	8.2922	0.840	9.871	0.843

Constant = −1,378.35
Std. err. of est. = 1,164.77 cu ft
Mean cu ft growth = 4,139.73 cu ft
Coef. var. = 28.1 per cent
Coef. of mul. det. = .622

states; this equation summarized in table 5 shows several important features of the inherent relationships that were not apparent using only simple regressions.

Cubic-foot volume growth responds to age in this equation, with basal area and site constant, in a manner opposite to that shown by the simple regression, i.e., volume growth now decreases with increasing age. Despite the low correlation of age and volume growth, age in this multiple equation is shown to be more effective in resolution of the growth variation than is site index, a higher Beta coefficient. Improvement of the prediction mechanism is made by using the three variable quadratic, SE_{est} = 1009.0 cu ft, R^2 = .721; cubic model, SE_{est} = 939.6 cu ft, R^2 = .762. Improvement over the linear model (table 5) is such that this complex equation would be justified. However, further investigations involving the linear and quadratic equations after transformation of the dependent and independent variables to logarithmic form indicated a further improvement of the predictive capacity. The equation selected for use in calculation of stand cubic-foot growth is shown in table 6; the linear model using logarithmic transformations of the variables.

Regression coefficients calculated for the independent variables are all highly significant as judged by the t-value and compared with $t_{.99\alpha}$. Site index, while it is the least valuable in resolution of the variation in the logarithm of cubic-foot growth, cannot be removed from the regression either on statistical or intuitive grounds, as occurred in the study of basal area growth. The quadratic logarithmic model showed only a slight improvement over the linear model with a SE_{est} of .09451 and R^2 of .8002; this improvement is minor and use of this more complex equation was not felt to be justified. Stand cubic-foot volume growth rates are shown in table 7 and figure 4.

Table 6. Prediction Equation for Net 10-Year Cubic-Foot Stand Volume Growth for Trees Larger than 4.5 Inches DBH

Variables	Reg. coef.	Std. error	t	Beta coef.
Log age.....	−.611509	.0542	−11.28	−.6955
Log BA.....	.760963	.0436	17.44	1.1652
Log site.....	.683943	.1486	4.60	.1967

Constant..... 1.16403
Std. err. of est. .. .09502 = 815.07 cu ft
Mean cu ft growth 3.57118 = 3,725.5 cu ft
Coef. var..... 21.93 per cent
Coef. of mul. det. .7954

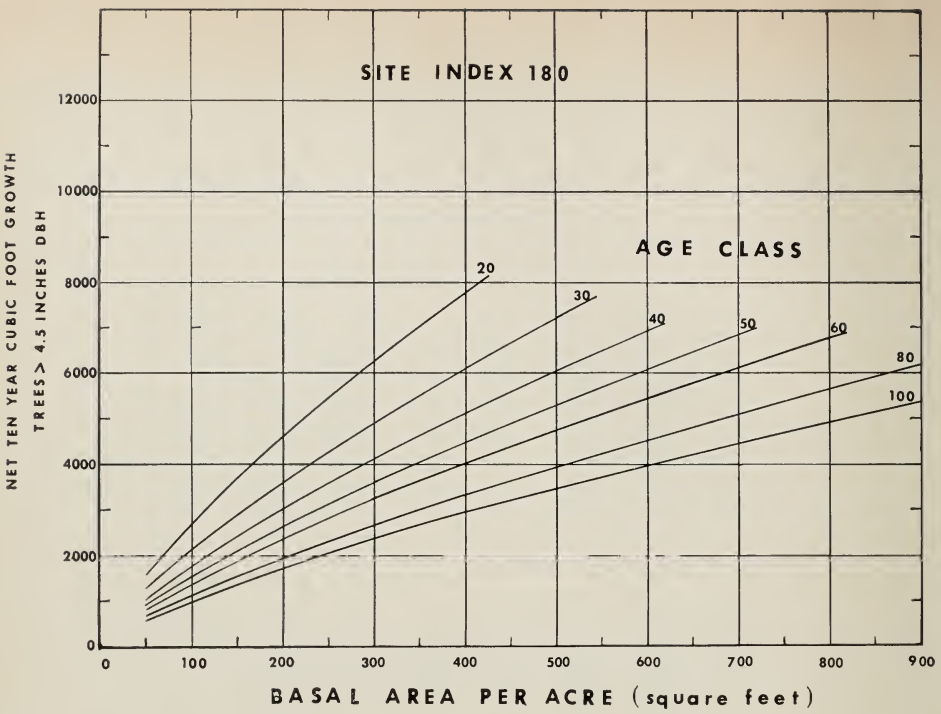
Maximum cubic-foot volume growth for all basal area classes occurs in the ten-year period following ten years of age. This may be the result of the use of the minimum diameter class of 4.5 inches at breast height; most trees will surpass this threshold diameter during the 10- to 20-year period and the high growth rates reflect this ingrowth. Growth drops rather abruptly between 20 and 30 years of age and begins a more gradual decline over the remaining time up to 100 years. Periodic growth subsequent to 20 years is perhaps relatively unaffected by stem ingrowth except in the lower site classes or those heavily overstocked. Periodic growth has not reached a maximum at 1,000 square feet of basal area for any site class at 100 years, meaning that added initial stand basal area continues to increase rate of volume growth. Extrapolation beyond 1,000 square feet of initial basal area at 100 years of age for all site classes indicates that the maximum growth occurs at basal area levels much larger than that normally found. In considering an actual stand, the increase of basal area with age

Table 7. NET 10-YEAR CUBIC-FOOT VOLUME STAND GROWTH FOR TREES OF ALL SPECIES LARGER THAN 4.5 INCHES AT BREAST HEIGHT

Age	Basal area per acre (sq ft)									
	50	100	200	300	400	500	600	700	800	900
	Volume growth (cu ft)									
years	Site Index 100									
10	1,634	2,769	4,692							
20	1,069	1,812	3,071	4,181						
30	834	1,414	2,396	3,263	4,061	4,813				
40	700	1,186	2,010	2,736	3,406	4,037	4,637			
50	610	1,034	1,753	2,387	2,972	3,522	4,046	4,549		
60	546	925	1,568	2,135	2,658	3,150	3,619	4,069	4,505	
70	497	842	1,427	1,943	2,419	2,867	3,293	3,703	4,099	4,484
80	458	776	1,315	1,791	2,229	2,642	3,035	3,413	3,778	4,132
90	426	722	1,224	1,666	2,074	2,458	2,824	3,176	3,515	3,845
100	399	677	1,147	1,562	1,945	2,305	2,648	2,977	3,296	3,605
	Site Index 120									
10	1,851	3,136	5,315							
20	1,211	2,053	3,479	4,736						
30	945	1,602	2,715	3,696	4,601	5,452				
40	793	1,343	2,277	3,100	3,858	4,573	5,253			
50	691	1,172	1,986	2,704	3,366	3,989	4,583	5,154		
60	618	1,048	1,777	2,419	3,011	3,568	4,100	4,610	5,103	
70	563	954	1,617	2,201	2,740	3,247	3,731	4,195	4,644	5,079
80	519	879	1,490	2,029	2,525	2,993	3,438	3,866	4,280	4,681
90	482	818	1,386	1,888	2,350	2,785	3,199	3,597	3,982	4,356
100	452	767	1,300	1,770	2,203	2,611	3,000	3,373	3,734	4,084
	Site Index 140									
10	2,056	3,485	5,906	8,041						
20	1,346	2,281	3,866	5,263	6,551					
30	1,050	1,780	3,017	4,107	5,112	6,059				
40	881	1,493	2,530	3,444	4,288	5,081	5,837			
50	768	1,302	2,207	3,005	3,741	4,433	5,093	5,727		
60	687	1,165	1,974	2,688	3,346	3,965	4,555	5,122	5,670	
70	625	1,060	1,797	2,446	3,045	3,609	4,146	4,662	5,160	5,644
80	576	977	1,656	2,254	2,806	3,326	3,821	4,296	4,756	5,202
90	536	909	1,541	2,098	2,611	3,094	3,555	3,997	4,425	4,840
100	503	852	1,444	1,967	2,448	2,901	3,333	3,748	4,149	4,538
	Site Index 160									
10	2,253	3,818	6,471	8,810						
20	1,475	2,499	4,235	5,766	7,178					
30	1,151	1,950	3,305	4,500	5,601	6,638				
40	965	1,636	2,772	3,774	4,698	5,567	6,396			
50	842	1,427	2,418	3,292	4,098	4,857	5,580	6,274		
60	753	1,276	2,163	2,945	3,666	4,344	4,991	5,612	6,213	
70	685	1,161	1,968	2,680	3,336	3,954	4,542	5,107	5,654	6,184
80	631	1,070	1,814	2,470	3,074	3,644	4,186	4,707	5,210	5,699
90	587	996	1,688	2,298	2,861	3,390	3,895	4,380	4,848	5,303
100	551	934	1,583	2,155	2,682	3,179	3,652	4,106	4,546	4,972

Table 7—Continued

Age	Basal area per acre (sq ft)									
	50	100	200	300	400	500	600	700	800	900
	Volume growth (cu ft)									
years	Site Index 180									
10	2,442	4,139	7,014	9,549						
20	1,598	2,709	4,591	6,250	7,780					
30	1,247	2,114	3,582	4,877	6,071	7,195				
40	1,046	1,773	3,004	4,091	5,092	6,034	6,932			
50	912	1,547	2,621	3,569	4,442	5,264	6,048	6,801		
60	816	1,383	2,345	3,192	3,974	4,709	5,410	6,083	6,734	
70	743	1,259	2,134	2,905	3,616	4,285	4,923	5,536	6,128	6,703
80	684	1,160	1,966	2,677	3,332	3,949	4,537	5,102	5,648	6,177
90	637	1,079	1,830	2,491	3,101	3,675	4,222	4,747	5,255	5,748
100	597	1,012	1,715	2,336	2,907	3,445	3,958	4,451	4,927	5,389
	Site Index 200									
10	2,625	4,448	7,538	10,263						
20	1,718	2,911	4,934	6,717	8,361					
30	1,340	2,272	3,850	5,242	6,525	7,733				
40	1,124	1,905	3,229	4,396	5,472	6,485	7,450			
50	981	1,662	2,817	3,835	4,774	5,658	6,500	7,309		
60	877	1,487	2,520	3,431	4,270	5,061	5,814	6,538	7,237	
70	798	1,353	2,293	3,122	3,886	4,606	5,291	5,950	6,586	7,204
80	736	1,247	2,113	2,877	3,581	4,244	4,876	5,483	6,070	6,639
90	684	1,160	1,966	2,677	3,333	3,949	4,537	5,102	5,648	6,177
100	642	1,088	1,844	2,510	3,125	3,703	4,254	4,784	5,295	5,792
	Site Index 220									
10	2,801	4,748	8,046	10,954	13,635					
20	1,833	3,107	5,266	7,170	8,924	10,576				
30	1,431	2,425	4,110	5,595	6,964	8,253	9,482			
40	1,200	2,034	3,447	4,692	5,841	6,922	7,952	8,942		
50	1,203	1,774	3,007	4,094	5,096	6,039	6,938	7,801	8,636	
60	1,076	1,587	2,690	3,662	4,558	5,402	6,206	6,978	7,725	8,449
70	979	1,444	2,448	3,332	4,148	4,916	5,647	6,350	7,030	7,689
80	902	1,331	2,256	3,071	3,823	4,530	5,205	5,852	6,478	7,086
90	839	1,238	2,099	2,858	3,557	4,215	4,843	5,446	6,028	6,594
100	787	1,161	1,968	2,679	3,335	3,952	4,541	5,106	5,652	6,182
	Site Index 240									
10	2,973	5,039	8,539	11,626	14,471					
20	1,946	3,298	5,589	7,609	9,471	11,224				
30	1,518	2,574	4,362	5,938	7,391	8,760	10,063			
40	1,273	2,158	3,658	4,980	6,199	7,346	8,440	9,490		
50	1,111	1,883	3,191	4,345	5,408	6,409	7,363	8,280	9,165	
60	994	1,684	2,855	3,886	4,838	5,733	6,586	7,406	8,198	8,967
70	904	1,533	2,598	3,537	4,402	5,217	5,994	6,740	7,461	8,160
80	833	1,413	2,394	3,259	4,057	4,808	5,524	6,211	6,876	7,520
90	775	1,314	2,228	3,033	3,775	4,474	5,140	5,780	6,398	6,998
100	727	1,232	2,089	2,844	3,540	4,195	4,819	5,419	5,999	6,561



4. Net ten-yer cubic-foot volume growth for all trees larger than 4.5 inches DBH, site index 180 for initial age and basal area classes.

means that for each prediction period, the stand will lie on a different basal area curve. Since as these stands become older, and the rate of basal area increment decreases, maximization of cubic-foot growth will occur as a result of increasing age long before 1,000 square feet of basal area is attained. The overall implication of these growth tables is that, disregarding possible effects of heavy stand density on height growth and stand health, adding basal area to stands improves the rate of cubic-foot volume growth. The difference in growth rate resulting from increasing site index is apparent, as age increases the influence of site becomes less, and site index lines are more closely spaced. Holding age constant as basal area increases shows that the site index lines diverge; this indicates that, increasing intial basal area, a fixed amount has a greater effect on volume growth as site index improves.

Board-foot volume growth (trees larger than 10.5 inches DBH)

Stand and site growth potential in terms of a board-foot unit provides volume predictions more easily translated into economic terms. Most transactions dealing with second-growth redwood are carried out with the board-foot unit as the medium of economic evaluation. Stand growth in this volumetric unit is influenced by the same set of conditions discussed for cubic-foot growth. Board-foot volume does not indicate total amount of wood present in a log, tree, or stand; it re-

flects amount of sawn wood recoverable, given certain adjustment features peculiar to the log rule on which the volume table is based. These adjustments deal with size of kerf, amount of slab, and dimensions of boards used to establish the rule. Because these adjustments have been different over a period of time and in different parts of the country, there are widely divergent scales of a log of given dimensions, depending on the log rule employed. The International ¼-inch rule is often accepted for dealing with problems of volume growth because of its recognition of inherent stem taper through use of short log sections. Widespread use of this log rule is not at the present time done in the redwood region where the dependence has been on the Scribner and Spaulding rules. Differences between stem volumes of Spaulding and International scaled trees becomes progressively smaller as the diameter increases (appendix D). Application of growth values developed by prediction equations requires means of conversion to express growth in terms of either the Spaulding or Scribner rules (appendix D).

The basic relationship of board-foot and cubic-foot growth are similar; growth is a function of diameter and height change of the individual trees in a sample. Some differences do occur which are not related solely to the use of different volume units. Growth in board-foot units refers only to trees larger than 10.5 inches, opposed to a minimum diameter of 4.5 inches for the cubic-foot growth. Reference is made to a different stand of trees in some of the samples, particularly when stands are young or when radial growth is such that ingrowth is still occurring into the larger minimum diameter limit. The utilized top diameter of included trees is larger for the board-foot stand, thereby reducing the portion of the stem included in the calculation of total stem volume.

Patterns of periodic board-foot growth (of 162 sample plots) relative to independent variables used in evaluation of growth are shown in table 23. The simple correlation coefficients indicate that as age, site, and basal area increase, there is a corresponding increase in stand board-foot growth. In general, only terms of basal area and site index are reasonably well correlated with board-foot increment; highest simple correlation is with log of basal area. As in the case of cubic-foot growth, age is again poorly correlated with volume growth, though somewhat higher. Higher correlation of board-foot growth and age may be due to ingrowth beyond the 10.5 inch limit which occurs for a longer period of time (most trees have exceeded the 4.5 inch limit by 10 years of age). Simple correlation coefficients support the premise that volume growth is well correlated with the height of stands as represented by site index. Initial stand basal area of trees larger than 4.5 inches has the highest correlation to this stand growth

Table 8. Linear Regression Equation of Net 10-Year Board-Foot Growth of Trees Larger than 10.5 Inches DBH (Int. 1/4)

Variant	Reg. coef.	Std. error	*t*	Beta coef.
Age	−208.620	37.308	−5.592	−.377
Site	207.647	26.555	7.819	.378
Basal area	55.103	4.880	11.291	.828

Constant	=	−18,793.8
Std. err. of est.	=	6,766.4 bd ft
Mean bd ft growth	=	26,213.0 bd ft
Coef. var.	=	25.8 per cent
Coef. of mul. det.	=	.722

measurement of any of the basic variables tested. Somewhat more than 50 per cent of the initial variation of stand board-foot growth is explained by this single measure of stand density.

The multiple regression equation using basic forms of age, site index, and basal area results in an adjustment of effects on board-foot growth not foreseeable when considering only simple relationships. This equation is summarized in table 8. The net regression lines depict the change in growth related to each of the variables while the other two variables are held constant, at their mean value. Increasing site index and initial stand basal area continues to result in the increase of board-foot growth. However, the effect of increasing age reverses and growth now drops with increasing age. Regarding the simple correlations of volume growth and age, the apparent increase in volume growth as age increases, is influenced by increasing stand basal area over time. The multiple regression equation permits fixing basal area and site index, thereby revealing that, as age increases, volume growth decreases. This facet of these complex reactions more satisfactorily agrees with the hypothesis of decreasing growth vigor with age. The masking of actual growth patterns, as seen in the simple regressions, can result in serious errors in evaluation of the growth process.

It became evident from further investigations of more complex equations that the interaction terms of the independent variables were of significant importance in the predictions of board-foot stand growth. The cubic model of the three basic variables does not provide sufficient improvement of the predictions over that of the quadratic model to warrant its use. The linear and quadratic models of logarithmic transformations of the dependent and independent variables indicated larger errors than those of the untransformed data, i.e., log, linear, ±10,091 board-feet; log quadratic ±6,290 board-feet. The larger standard errors and lower coefficients of multiple determination indicated that these logarithmic transformations were not the most suitable for approximations of board-foot growth in this segment of the stand.

The equation selected for the calculation of the board-foot growth values (table 10 and figure 5) is shown in table 9. This equation is based on the quadratic model; however, the term of the square of age has been omitted because its inclusion resulted in an increase of the standard error of estimate to 5,311 board-feet; also the *t*-value calculated for this term indicated that this regression coefficient was not significantly different than zero.

Use of the untransformed quadratic model of the data for expressing the net periodic board-foot growth results in a somewhat different shape of these growth curves than those of cubic-foot stand growth. In the board-foot growth component there is evident a more

Table 9. Prediction Equation for Net 10-Year Board-Foot Growth of Trees Larger than 10.5 Inches DBH (Int. 1/4)

Variant	Reg. coef.	Std. error	*t*	Beta coef.
Basal area^2	−.043467	.01608	−2.703	−.625
Site^2	.143483	.07531	1.905	.091
Basal area × age	−.243732	.13876	−1.756	−.419
Site × age	−.444020	.41999	−1.057	−.158
Basal area × site	.573365	.052867	10.845	1.739

Constant = 2,457.02
Std. err. of est. = 5,294.45 bd ft
Mean bd ft growth = 26,213.0 bd ft
Coef. var. = 20.19 per cent
Coef. of mul. det. = .827

rapid reduction of net growth rate as stand basal area increases. For the lower site classes (up to 140) maximum board-foot growth occurs prior to attaining 1,000 square feet of basal area; this occurs in all age classes. As site index increases maximum growth occurs at progressively higher basal

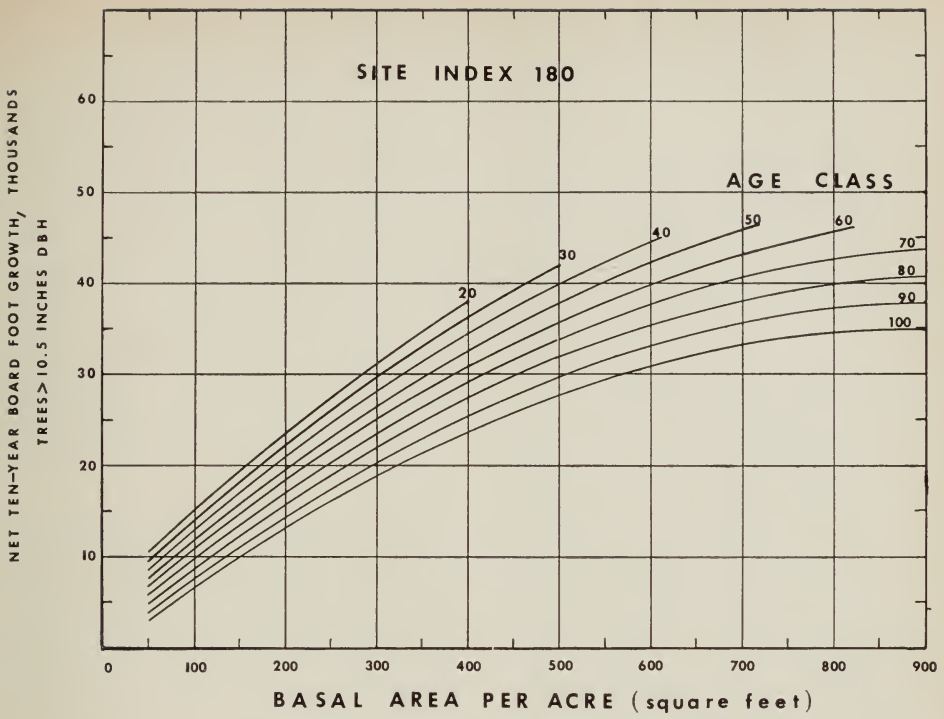
Table 10. Net 10-Year Board-Foot Volume Growth
(Trees larger than 10.5 inches DBH, Int. 1/4)

Age	Basal area per acre (sq ft)									
	50	100	200	300	400	500	600	700	800	900
	Volume growth (sq ft)									
years	Site Index 100									
20.....	5,518	7,815	11,757	14,830						
30.....	4,952	7,128	10,826	13,655	15,615					
40.....	4,386	6,440	9,895	12,480	14,196	15,043				
50.....	3,821	5,752	8,963	11,305	12,777	13,380	13,113			
60.....	3,255	5,064	8,032	10,129	11,358	11,717	11,207	9,828		
70.....	2,689	4,377	7,100	8,954	9,939	10,055	9,301	7,677	5,185	
80.....	2,123	3,689	6,169	7,779	8,520	8,392	7,394	5,527	2,791	
90.....	1,557	3,001	5,237	6,604	7,101	6,729	5,488	3,377	397	
100.....	991	2,313	4,306	5,429	5,682	5,066	3,581	1,227		
	Site Index 120									
20.....	6,545	9,416	14,505	18,724						
30.....	5,891	8,639	13,484	17,460	20,567					
40.....	5,236	7,863	12,464	16,196	19,059	21,052				
50.....	4,581	7,086	11,444	14,932	17,551	19,301	20,181			
60.....	3,927	6,310	10,424	13,668	16,043	17,549	18,186	17,953		
70.....	3,272	5,533	9,403	12,404	14.536	15,798	16,191	15,714	14,368	
80.....	2,617	4,756	8,383	11,140	13,028	14,046	14,195	13,475	11,886	9,427
90.....	1,962	3,980	7,363	9,876	11,520	12,295	12,200	11,236	9,403	6,700
100.....	1,308	3,203	6,342	8,612	10,012	10,543	10,205	8,997	6,920	3,974
	Site Index 140									
20.....	7,687	11,131	17,367	22,733						
30.....	6,944	10,266	16,258	21,380	25,633					
40.....	6,200	9,400	15,148	20,027	24,037	27,177				
50.....	5,457	8,535	14,039	18,674	22,440	25,337	27,364			
60.....	4,713	7,670	12,930	17,322	20,844	23,496	25,280	26,194		
70.....	3,970	6,804	11,821	15,969	19,247	21,656	23,196	23,866	23,667	
80.....	3,226	5,939	10,712	14,616	17,651	19,816	21,112	21,538	21,095	19,783
90.....	2,483	5,073	9,603	13,263	16,054	17,975	19,028	19,210	18,524	16,968
100.....	1,739	4,208	8,494	11,910	14,457	16,135	16,944	16,883	15,952	14,153
	Site Index 160									
20.....	8,944	12,961	20,343	26,856	32,500					
30.....	8,112	12,007	19,145	25,415	30,815	35,345				
40.....	7,279	11,053	17,948	23,973	29,129	33,416	36,834			
50.....	6,447	10,099	16,750	22,532	27,444	31,487	34,661	36,965		
60.....	5,615	9,144	15,552	21.090	25,759	29,558	32,488	34,549	35,740	
70.....	4,782	8,190	14,354	19,648	24,073	27,529	30,315	32,132	33,080	33,158
80.....	3,950	7,236	13,156	18,207	22,388	25,700	28,142	29,716	30,420	30,254
90.....	3,118	6,282	11,958	16,765	20,703	23,771	25,970	27,299	27,759	27,350
100.....	2,285	5,328	10,760	15,323	19,017	21,842	23,797	24,883	25,099	24,446

Table 10—Continued

Age	Basal area per acre (sq ft)									
	50	100	200	300	400	500	600	700	800	900
	Volume growth (sq ft)									
years	Site Index 180									
20	10,315	14,906	23,435	31,095	37,885					
30	9,394	13,863	22,148	29,564	36,111	41,788				
40	8,473	12,820	20,862	28,034	34,337	39,770	44,335			
50	7,552	11,777	19,575	26,503	32,563	37,752	42,073	45,524		
60	6,631	10,734	18,288	24,973	30,788	35,735	39,811	43,019	45,357	
70	5,710	9,691	17,001	23,442	29,014	33,717	37,550	40,513	42,608	43,833
80	4,789	8,648	15,715	21,912	27,240	31,699	35,288	38,008	39,859	40,840
90	3,868	7,605	14,428	20,382	25,466	29,681	33,026	35,503	37,110	37,847
100	2,946	6,562	13,141	18,851	23,692	27,663	30,765	32,997	34,360	34,854
	Site Index 200									
20	11,802	16,965	26,641	35,448	43,385	50,453				
30	10,792	15,834	25,266	33,828	41,522	48,346	54,301			
40	9,782	14,702	23,890	32,209	39,659	46,239	51,950	56,792		
50	8,772	13,570	22,515	30,590	37,796	44,133	49,600	54,198	57,926	
60	7,762	12,438	21,139	28,971	35,933	42,026	47,249	51,603	55,088	57,704
70	6,752	11,307	19,764	27,352	34,070	39,919	44,899	49,009	52,250	54,622
80	5,742	10,175	18,388	25,732	32,207	37,812	42,548	46,415	49,412	51,541
90	4,732	9,043	17,013	24,113	30,344	35,706	40,198	43,821	46,575	48,459
100	3,722	7,911	15,637	22,494	28,481	33,599	37,848	41,227	43,737	45,377
	Site Index 220									
20	13,403	19,140	29,962	39,916	48,999	57,214				
30	12,304	17,919	28,498	38,208	47,048	55,018	62,120			
40	11,205	16,699	27,034	36,499	45,096	52,823	59,681	65,669		
50	10,106	15,478	25,569	34,791	43,144	50,627	57,241	62,986	67,861	
60	9,008	14,257	24,105	33,083	41,192	48,432	54,802	60,303	64,935	68,697
70	7,909	13,037	22,641	31,375	39,241	46,236	52,363	57,620	62,008	65,526
80	6,810	11,816	21,177	29,667	37,289	44,041	49,924	54,937	59,081	62,356
90	5,712	10,596	19,712	27,959	35,337	41,845	47,484	52,254	56,154	59,185
100	4,613	9,375	18,248	26,251	33,385	39,650	45,045	49,571	53,228	56,015
	Site Index 240									
20	15,118	21,429	33,398	44,498	54,729	64,090				
30	13,931	20,120	31,845	42,701	52,688	61,806	70,054			
40	12,743	18,810	30,292	40,904	50,648	59,521	67,526	74,661		
50	11,556	17,501	28,739	39,108	48,607	57,237	64,998	71,889	77,911	
60	10,368	16,191	27,186	37,311	46,566	54,953	62,470	69,117	74,896	79,805
70	9,181	14,882	25,633	35,514	44,526	52,668	59,942	66,346	71,880	76,545
80	7,993	13,573	24,080	33,717	42,485	50,384	57,414	63,574	68,865	73,286
90	6,806	12,263	22,526	31,920	40,445	48,100	54,886	60,802	65,849	70,027
100	5,618	10,954	20,973	30,123	38,404	45,816	52,358	58,030	62,834	66,768

area levels until for site index 240 the maximization of growth, for all ages, is beyond 1,000 square feet. The importance of this is that concentration of effort to improve the basal area level of stands is most effectively utilized on better sites. For most site and age classes the basal area level at which this point of maximum board-foot growth is reached is perhaps beyond



5. Net ten-year board-foot volume growth for all trees larger than 10.5 inches DBH, site index 180 for initial age and basal area classes (basal area for all trees larger than 4.5 inches).

the stocking level that can normally be expected to be reached in these young-growth stands of redwood. Furthermore, in the denser classes the marginal gain in board-foot growth, realized by additional units of stand basal area, is perhaps not worth the added cost of increasing stand basal area.

Regarding the fixed basal area classes, maximum board-foot growth occurs in the 10- to 20-year period. Subsequently growth drops for these fixed basal area classes in a linear manner; the rate of decrease becomes larger as both site and basal area increase. The effect of increasing age and perhaps some loss in growth vigor with age does not permit the display of the increase in stand growth that may be attributed to stem ingrowth beyond the 10.5 inch diameter limit. What the high initial growth values in the 10- to 20-year period does indicate is that a majority of the stems surpasses the threshold diameter limit during this period. It should be recognized, particularly in the lower site classes, that a portion of the stand growth does result from stem ingrowth over a period of time after 20 years of age. It is perhaps also true that this period of time, over which ingrowth is a factor, is extended as the initial basal area becomes greater as a result of a decrease in the radial increment of stems with increasing density. It should be kept in mind that three components of stand growth are in operation in these figures—survivor growth, ingrowth, and mortality. The general pattern of decreas-

ing net stand growth over the range of initial ages indicates that loss of radial growth vigor is not offset by stem ingrowth.

Board-foot volume growth (trees larger than 15.5 inches DBH)

A discussion of the board-foot growth of the segment of stands which have exceeded 15.5 inches DBH is relevant to this discussion of stand growth, because it more closely depicts the portion of stands in which management is primarily interested. Trees of this size are considerably closer to harvest, and they represent approximately the minimum standard of utilization currently in operation in this region. For these reasons, this stand segment is of more practical interest to owners of young-growth stands.

A summary of the simple correlation coefficients of regressions of board-foot growth of trees over 15.5 inches DBH and the list of independent variables is shown in table 23. The small improvement of the correlation between this board-foot growth and age over that of the stand larger than 10.5 inches may be due to the effect of the increased period of time over which ingrowth occurs. The more strongly defined upward trend of volume growth over age results from the longer period of time required for trees to surpass the minimum diameter limit established for this stand segment. Furthermore, growth rate is now more strongly influenced by ingrowth of a single tree, since its initial contribution to the stand is larger than that of the 10.5 inch tree. Site index is reasonably well correlated with this board-foot volume growth; the trend of data indicates a fairly linear trend, and explains approximately 40 per cent of the original variation in growth. The improvement of volume growth with site index is consistent with past experience in study of stand volume growth. Correlation of age and site is low; this is desirable because it indicates that the sampling did not consistently select stands on which the age increased with site index. Volume growth reacts to stand basal area in a manner similar to that of the cubic- and the smaller diameter board-foot studies described earlier. The highest simple correlation coefficients are again associated with independent terms which contain the basal area variable.

The multiple regression model with the basic forms of age, site, and basal area, shown in table 11, indicates that this form of equation is more successful in explanation of the stand growth than any of the combined or transformed variables tested in the simple regression format. The simple correlation coefficients indicate that growth increases with each of the independent variables. Holding two variables constant and solving for the third points out an interaction not previously indicated. Increasing age in this equation results in a decline in stand growth; site and basal area retain their effect on growth with some modification in the degree of effect. This linear equation

Table 11. Linear Equation of Net 10-Year Board-Foot Volume Growth of Trees Larger than 15.5 Inches DBH (Int. 1/4)

Variable	Reg. coef.	Std. error	t	Beta coef.
Age	−182.0300	44.8523	−4.058	−.301
Site	207.952	31.9347	6.511	.354
Basal area	55.6147	5.8522	9.503	.766

Constant = −21,216.5
Std. err. of est. = 8,078.0 bd ft
Mean bd ft growth = 25,206.3 bd ft
Coef. of var. = 32.04 per cent
Coef. of mul. det. = .649

Table 12. Prediction Equation for Net 10-Year Board-Foot Volume Growth for Trees Larger than 15.5 Inches DBH (Int. 1/4)

Variable	Reg. coef.	Std. error	t	Beta coef.
Basal area^2....	−.0070843	.02076	− .341	−.094
Site^2.....	.117729	.09896	1.189	.069
Basal area × age.....	−.616393	.18086	−3.408	−.979
Site × age....	.792339	.55019	1.440	.257
Basal area × site.....	.502870	.06871	7.318	1.394

Constant = − 1,965.14
Std. err. of est. = 6,805.38 bd ft
Mean bd ft growth = 25,206.3 bd ft
Coef. of var. = 26.99 per cent
Coef. of mul. det. = .752

indicates that the growth of this segment of the stands is more variable than the stand larger than 10.5 inches in diameter. The mean stand growth is less, yet the SE_{est} is larger than that of the linear equation of the stand with the 10.5 inch diameter limit. This equation, as in other volume growth equations, shows a reversal in the effect of stand age over that of the simple growth-age relationship; stand growth in this model decreases with increasing age.

Patterns of results in investigations of the models tested follow those of the stand larger than 10.5 inches. The logarithmic transformations of the data did not reduce the SE_{est} to the levels found in the equations of the untransformed variables. The cubic model of the data in the untransformed state did not have a materially improved SE_{est} over that of the quadratic model. Generally, values of the standard errors of estimate of comparable models in this larger diameter limit were higher than those of the smaller diameter stand. The quadratic model, with age squared removed, was selected for preparation of the growth tables since its SE_{est} was nearly at the minimum value of the models tested. Of more importance for the use of these equations is that predictions of the two stand segments of board-foot growth are made from the same independent terms, only the regression coefficients differ. Prediction of growth and preparation of the growth values (table 13 and figure 6) was made from the equation shown in table 12.

In this model, stand growth, except for site indices 140 and less, reaches its maximum at approximately 1,000 square feet of basal area. For the lower site classes the basal area level at which an age class reaches its maximum growth is reduced as the age increases. Maximum stand growth for site class 100 for ages 80 through 100 years occurs at 100 square feet of basal area; indicating that for this low site index the effect of stem ingrowth may extend well into the older ages. The change in growth for a given age and site class across the basal area classes is nearly linear, but tending to drop as basal area increases. For the basal area class 100 it is noted that growth increases with increasing age in all sites. In basal area class 200 for sites 100 to 140 the growth decreases with increasing age, for sites 160 and better the growth increases with age. Growth for all other basal area classes regardless of site index, decreases with increasing stand age; with the rate of decrease becoming greater as the initial stand basal area becomes larger.

Comparison of growth curves for the age, site, and basal area classes of the 10.5 and 15.5 inch stand segments indicates that in the younger ages the growth of the smaller diameter stands exceeds that of the larger segment; this occurs within the basal area classes that

Table 13. Net 10-Year Board-Foot Volume Growth
(Trees larger than 15.5 inches DBH, Int. 1/4)

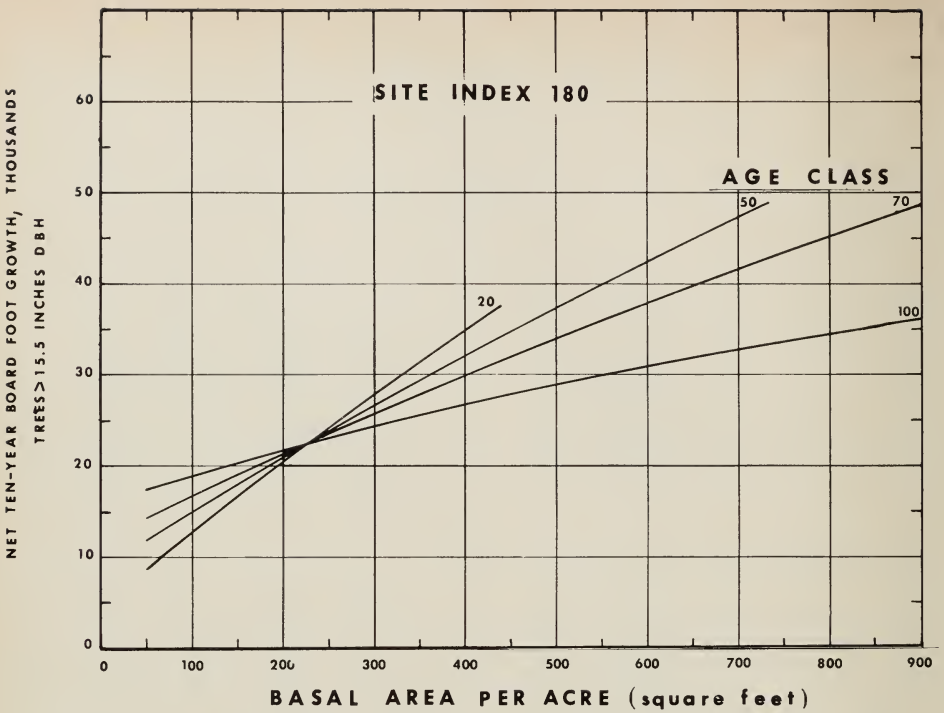
Age	Basal area per acre (sq ft)									
	50	100	200	300	400	500	600	700	800	900
	Volume growth (sq ft)									
years	Site Index 100									
20	2,677	4,522	8,105	11,547						
30	3,161	4,698	7,665	10,490	13,174					
40	3,645	4,874	7,224	9,433	11,501	13,426				
50	4,130	5,050	6,784	8,376	9,827	11,136	12,304			
60	4,614	5,226	6,343	7,320	8,154	8,847	9,398	9,807		
70	5,098	5,402	5,903	6,263	6,481	6,557	6,492	6,285	5,936	
80	5,582	5,578	5,463	5,206	4,808	4,268	3,586	2,762	1,797	691
90	6,066	5,754	5,022	4,149	3,134	1,978	680			
100	6,550	5,929	4,582	3,092	1,461					
	Site Index 120									
20	4,015	6,363	10,952	15,399						
30	4,657	6,697	10,670	14,501	18,190					
40	5,300	7,031	10,388	13,602	16,675	19,607				
50	5,943	7,366	10,106	12,704	15,161	17,475	19,649			
60	6,585	7,700	9,824	11,806	13,646	15,344	16,901	18,316		
70	7,228	8,035	9,542	10,907	12,131	13,213	14,154	14,952	15,609	16,125
80	7,871	8,369	9,260	10,009	10,616	11,082	11,406	11,588	11,629	11,528
90	8,513	8,703	8,978	9,111	9,102	8,951	8,658	8,224	7,649	6,931
100	9,156	9,038	8,696	8,212	7,587	6,820	5,911	4,860	3,668	2,334
	Site Index 140									
20	5,447	8,297	13,892	19,345						
30	6,248	8,790	13,769	18,606	23,301					
40	7,049	9,283	13,645	17,866	21,944	25,881				
50	7,850	9,776	13,522	17,126	20,588	23,909	27,088			
60	8,651	10,269	13,398	16,386	19,232	21,936	24,499	26,919		
70	9,452	10,762	13,275	15,646	17,875	19,963	21,909	23,714	25,371	26,898
80	10,253	11,255	13,151	14,906	16,519	17,991	19,320	20,508	21,555	22,460
90	11,054	11,748	13,028	14,166	15.163	16,018	16,731	17,303	17,733	18,021
100	11,856	12,241	12,904	13,426	13,807	14,045	14,142	14,097	13,911	13,583
	Site Index 160									
20	6,973	10,326	16,927	23,386						
30	7,933	10,978	16,962	22,805	28,505					
40	8,892	11,629	16,997	22,223	27,308	32,250				
50	9,852	12,281	17,032	21,642	26,110	30,436	34,621			
60	10,811	12,932	17,067	21,060	24,912	28,622	32,190	35,617		
70	11,771	13,583	17,102	20,479	23,714	26,808	29,760	32,570	35,238	37,765
80	12,730	14,235	17,187	19,897	22,516	24,993	27,329	29,523	31,575	33,485
90	13,690	14,886	17,172	19,316	21,318	23,179	24,898	26,476	27,911	29,206
100	14,649	15,537	17,207	18,735	20,121	21,365	22,468	23,429	24,248	24,926

can be reasonably expected. As site increases, with basal area held constant, the age becomes less at which the larger stand growth exceeds that of the smaller stand. This reversal in growth rates, the larger stand exceeding the smaller stand, is not unexpected and is a function of the use of different mini-

Table 13—Continued

Age	Basal area per acre (sq ft)									
	50	100	200	300	400	500	600	700	800	900
	Volume growth (sq ft)									
years	Site Index 180									
20	8,593	12,450	20,056	27,521	34,844					
30	9,711	13,260	20,249	27,098	33,804	40,369				
40	10,829	14,069	20,443	26,675	32,765	38,713	44,520			
50	11,947	14,879	20,636	26,252	31,726	37,058	42,248	47,297		
60	13,065	15,689	20,830	25,829	30,686	35,402	39,976	44,408	48,699	52,848
70	14,183	16,499	21,023	25,406	29,647	33,746	37,704	41,520	45,194	48,727
80	15,302	17,309	21,217	24,983	28,608	32,090	35,432	38,631	41,689	44,605
90	16,420	18,118	21,410	24,560	27,568	30,435	33,160	35,743	38,184	40,484
100	17,538	18,928	21,603	24,137	26,529	28,779	30,887	32,854	34,679	36,363
	Site Index 200									
20	10,308	14,667	23,279	31,750	40,078					
30	11,584	15,635	23,631	31,485	39,197	46,768				
40	12,861	16,604	23,983	31,221	38,317	45,271	52,083			
50	14,137	17,572	24,335	30,956	37,436	43,773	49,970	56,024		
60	15,414	18,540	24,687	30,692	36,555	42,276	47,856	53,294	58,590	63,745
70	16,690	19,509	25,039	30,427	35,674	40,779	45,742	50,564	55,244	59,782
80	17,967	20,477	25,391	30,163	34,793	39,282	43,629	47,834	51,897	55,819
90	19,243	21,445	25,742	29,898	33,912	37,784	41,515	45,104	48,551	51,857
100	20,520	22,413	26,094	29,634	33,031	36,287	39,401	42,374	45,205	47,894
	Site Index 220									
20	12,117	16,979	26,597	36,073	45,407					
30	13,552	18,105	27,107	35,967	44,685	53,261				
40	14,987	19,232	27,617	35,861	43,962	51,922	59,741			
50	16,422	20,359	28,128	35,755	43,240	50,583	57,785	64,846		
60	17,856	21,486	28,638	35,649	42,517	49,245	55,830	62,274	68,576	74,737
70	19,291	22,613	29,148	35,543	41,795	47,906	53,875	59,702	65,388	70,932
80	20,726	23,739	29,659	35,437	41,073	46,567	51,920	57,131	62,200	67,128
90	22,161	24,866	30,169	35,330	40,350	45,228	49,964	54,559	59,012	63,323
100	23,596	25,993	30,679	35,224	39,628	43,889	48,009	51,988	55,824	59,519
	Site Index 240									
20	14,020	19,385	30,008	40,490	50,830					
30	15,613	20,670	30,677	40,542	50,266	59,848				
40	17,206	21,955	31,346	40,595	49,702	58,668	67,492			
50	18,800	23,240	32,015	40,647	49,138	57,488	65,695	73,761		
60	20,393	24,525	32,683	40,700	48,574	56,307	63,898	71,348	78,656	85,822
70	21,987	25,811	33,352	40,752	48,010	55,127	62,102	68,935	75,626	82,176
80	23,580	27,096	34,021	40,805	47,446	53,947	60,305	66,522	72,597	78,530
90	25,174	28,381	34,690	40,857	46,882	52,766	58,508	64,109	69,567	74,884
100	26,767	29,666	35,359	40,909	46,318	51,586	56,712	61,695	66,538	71,238

mum diameter limits. Setting these diameter limits indicates that as long as trees can enter the net stand growth values as ingrowth they will continue to cause some disruption of the growth patterns. The initial contribution of these larger trees to stand growth, regardless of when they enter from the



6. Net ten-year board-foot volume growth for all trees larger than 15.5 inches DBH, site index 180 for initial age and basal area classes (basal area for all trees larger than 4.5 inches).

smaller diameter limit stand, is going to be large. For example, a stem which surpasses the 15.5 inch limit between 30 and 40 years contributes to the 10.5 inch stand growth only its volume increment during the period, while to the growth of the stand larger than 15.5 inches it contributes its entire accumulated volume. Growth values assume a degree of equality in a portion of the tables; this condition perhaps reflects the point when all stems have exceeded the 15.5 diameter limit. Extending the curves beyond this range of compatibility the smaller stand has a tendency to drop off at a more rapid rate than for the larger diameter segment. This may be due to an increase in mortality of the stems between 10.5 and 15.5 inches. It seems reasonable to consider that the more dense stands will have greater losses as the result of suppression in the smaller diameters.

APPLICATION OF EQUATIONS

The function of prediction equations is the preparation of growth tables for evaluation of future ten-year net periodic stand growth. Stand characteristics for predicting growth include basal area of trees of all species over 4.5 inches in diameter at breast height; average breast-high age and total height are evaluated from dominant redwood trees. Users applying these equations for predictions should be aware of several pertinent facts about the gen-

eral description of the character of stands for which the equations are recommended.

Descriptive statistics of the sampled redwood stands are covered in the textual material and sample distribution tables (appendix B). Selection of stands for sampling was directed toward securing a cross-section of stands which indicated a potential of producing an economically valuable crop. Application of equations to stands recently cut or showing effects of extensive blowdown is not recommended because of the release effects these influences may have on future stand growth. Two-storied stands, regardless of the cause, should not be used as the basis for prediction. This restriction refers to stands with a large number of the original stand residuals; effects of these trees are not included in the growth equations. A specific species composition is not indicated from the studies of the data; however, it is felt that equations are not appropriate for stands with less than 50 per cent redwood basal area. Age refers to the average breast-high age of dominant redwood; average stand age and average total height of dominant redwood are used to determine stand site index. Definition of dominant redwood as the basis for determination of site index, despite the presence of taller white-wood species, requires that dominance be considered only in terms of the redwood portion of the stand. This has a bearing on use in stands with at least 50 per cent redwood basal area; if redwood comprises less than this amount, expression of redwood dominance may be difficult to ascertain. For convenience and simplification of field measurements, all prediction equations are based on stand basal area for trees larger than 4.5 inches at breast height; this stand measurement is used for predictions of board-foot volumes of the 10.5 and 15.5 inch minimum stand diameter limits.

Net-growth values are expressions of expected stand periodic growth for fixed combinations of initial stand measurements. Regardless of basic stand characteristics in the prediction of growth, except for site index, there is a constant change of stand values as a result of current growth; this causes the prediction base to vary from year to year. Growth as shown does not depict the type of growth expected from a given stand over a long period of time, as do curves associated with yield tables.

Selection of the sample units and the measurement of the necessary predictive stand values requires careful consideration in the field. Predictions of future stand growth are highly variable under the best possible conditions and errors resulting from poor stem measurements can compound errors in prediction. As in all sampling, the number of sample units required to derive a prediction of the future growth of a stand depends on the variation of the stand, the level of accuracy required, and the probability level at which the predictions are to be made. The regression estimators described in this study refer to the relationship of the various defined components of stand growth to the independent variables of this particular sample of young-growth redwood. It is necessary to infer from the sample of the entire range of young-growth stands that growth values of this sample hold true for the total population. This inference creates problems in subsequent applications because we cannot state explicitly that subsequent samples of young redwood

stands will show similar growth relationships.

For application of these prediction equations it will be necessary to establish a method of sampling the stands under consideration. Choice of sampling unit is important. The fixed area plot is the conventional system; most foresters are familiar with its establishment and relationship of sample measurements to an acre basis. However, the point sample offers distinct advanages to users of these equations because of ease in securing a sample stand basal area. An important added consideration is that point samples were utilized for collection of the data from which these equations were derived. Of the variables for the implementation of the growth equations (age, site index, and basal area), only basal area would be evaluated on a different basis by the fixed area plot. Breast-high age and average total height of dominant redwood would be determined in the same manner regardless of the type of sample unit used. A recommendation for the type sampling unit to use must consider the degree of compatability between data used in derivation of the equations and their subsequent application when point sampling is employed to sample the stands being considered.

Stand growth determined by these equations from sample estimates of the basic predictive variables (age, site, and basal area) implies that a sampling error is associated with the predicted growth values. These sampling errors are in addition to the prediction errors resulting from the variation of the regression coefficients of the equations. The standard errors of the regression coefficients, shown in the equation tables, are an integral part of these empirical equations and nothing can be done about removing their effects from subsequent predictions of growth. It is possible, however, to reduce the sampling error by installation of more samples. Through a preliminary sample an estimate of the coefficient of variation of the growth component of the strata in question is determined. This estimate of the coefficient of variation permits some control over the subsequent sampling errors by indicating if more sample units are required to attain the particular level of confidence and allowable error. This process should not be considered as affecting errors of the prediction equations; regardless of how many additional sample units are established, the errors inherent in the equations remain unaffected. The level of prediction error for each equation is exemplified by the SE_{est}; this value lies generally between 20 and 30 per cent of the mean value of the growth. These relatively high values of regression error are partially the result of attempting to provide general growth equations for the entire range of stand conditions that currently exist in the young-growth stands.

Two approaches may be followed in application of the stand measurements to the prediction equations given for basal area and cubic foot volume growth. The most obvious is that of making a growth prediction for each stand observation. These results, a series of predicted growth values, can be used to determine the estimate of mean predicted stand growth and its variation. The second procedure entails making a single prediction of mean growth using the mean sample values of the independent terms in the prediction equation being considered. An example of these procedures is

shown in table 14. This example, based on 10 one-fifth acre remeasured sample plots, indicates predictions of the net 10-year stand growth for basal area and cubic-foot volume. It is shown that the logarithmic mean growth of the 10 plots is the same as the logarithmic mean growth evaluated from the mean values of the independent variables. It should be pointed out that, in the case of the nonlinear equations of board-foot growth, it is not correct to use the average values of the x's to make a single prediction of mean growth since this results in a biased estimate. The procedure for the evaluation of mean board-foot growth requires a prediction for each observation.

The preceding discussion of errors associated with the prediction process pointed out that variation of the regression coefficients is an integral aspect of the predictions systems that cannot be effectively eliminated. The standard error of estimate, SE_{est}, presented with each of the equations is the expression of the differences between measured and predicted stand growth and is an expression of the variation between the field estimate of growth and that estimate arrived at by the prediction equations. Establishment of a level of confidence about these prediction estimates, for the range of independent basic variables of this sample data, requires use of the standard error of estimate. This statement of confidence implies limits about these multispace predictions, developed by the equations, within which estimates of the true population growth lie. These limits may be computed for each entry

Table 14. Example of the Application of the Prediction of Basal Area and Cubic-Foot Net Periodic Growth of Trees Larger than 4.5 Inches DBH

Plot	Age	BA	Site Index	Pred. of Log growth	Anti-Log growth	Pred. of Log BA growth	Anti-Log BA growth
	years	*sq ft*			*cu ft*		*sq ft*
1	34	325	164	3.6537	4,505	2.0126	103
2	17	112	155	3.4689	2,944	2.0344	107
3	19	97	147	3.3763	2,379	1.9507	89
4	37	372	167	3.6813	4,801	2.0112	103
5	33	280	170	3.6230	4,198	1.9873	97
6	33	127	179	3.3771	2,383	1.7854	61
7	35	237	143	3.5009	3,163	1.9197	83
8	34	303	164	3.6305	4,271	1.9947	99
9	35	368	174	3.7047	5,066	2.0320	107
10	20	66	166	3.2714	1,868	1.8305	68
Totals	297	2,287	1,629	35.2878	35,578	19.5585	918
Average	29.7	228.7	162.9	3.52878	3,557.8	1.95585	91.8
Anti-log				3379.0		90.3	

Predictions using mean values of the independent variables

Basal area growth

Mean log of age = 1.4566 × −.975706 = −1.4212
Mean log of BA = 2.2911 × .587816 = 1.3467
Constant term = 2.0304

1.9559 = 90.3 sq ft

Cubic-foot volume growth

Mean log of age = 1.4566 × −.611509 = −.8907
Mean log of BA = 2.2911 × .760963 = 1.7434
Mean log of site = 2.2109 × .683943 = 1.5121
Constant term = = 1.1640

3.5288 = 3,379 cu ft

in the growth tables across the ranges of the basic independent variables included in the equations. As a matter of practical use, these limits may have little value to the user and the process of reporting these growth tables would be more involved. Of more importance is that the user be aware that as the fixed values of the independent variables move from their means, in this sample data, the width of the given confidence interval becomes greater. Development of these limits about the regression estimates may be done using the equation (Snedecor, 1956):

$$\hat{Y} \pm t \cdot SE_{est} \cdot \sqrt{\frac{1}{N} + C_{11}x_i^2 + C_{22}x_j^2 + 2C_{12}x_ix_j} \qquad (1)$$

where

C_{ij}, etc. = elements of the inverse matrix (multipliers)

x_i, x_j = difference between X_1 and X_2 from the mean values of these variables in the data $x_i = (x_i - \bar{x}_i)$

N = Number of samples used to develop regressions

$\hat{Y}$ = Predicted growth

The model shown in equation (1) represents computations for a regression equation with two independent variables. Increasing the number of predictive variables to three increases the number of values of "C" to six. Computation of confidence limits for a given probability level for the mean values of the independent variables of the regression data is simple since the values used for X_2 are zero. In this instance, with X_i fixed at $\bar{X}_i$, the use of zero with the "C" elements means that all computations that include a zero value can be ignored; equation (1) reduced to:

$$\hat{Y} \pm t \cdot SE_{est} \cdot \sqrt{1/N}$$

Substitution of the t value for a 95 per cent probability level (1.96), the computed values of the SE_{est}, and the number of sample units used in each of the prediction equations shows the following results:

BA gr. > 4.5″ $84.87 \pm (1.96 \times 27.53 \times \sqrt{1/163}) = \pm 4.23$ sq ft

Cu-ft gr. > 4.5″ $3725 \pm (1.96 \times 850.7 \times \sqrt{1/163}) = \pm 133.38$ cu ft

Bd-ft gr. > 10.5″ $25213 \pm (1.96 \times 5394.4 \times \sqrt{1/162}) = \pm 815.1$ bd ft

Bd-ft gr. > 15.5″ $25206 \pm (1.96 \times 6805.4 \times \sqrt{1/158}) = \pm 1061.2$ bd ft

These estimates of confidence limits refer specifically to the prediction equations data, in this case: average age, approximately 41 years; average site index, 175; and the average basal area, approximately 310 square feet. Stand basal area values differ slightly because of the different number of sample

units used for the various components of stand growth considered in this work.

To calculate an estimate of the variance of the predicted growth of a set of simple random sample plots established by the user, an adaptation of a technique described by Cunia (1965) is suggested. In this estimation of variance, the two items bearing on the source of errors in these predictions are considered. The original set of sample data, from which the prediction equations were derived, provides a matrix of "c multipliers" which are related to the errors associated with the regression coefficients; these matrices have been computed and presented in appendix F, tables 28, 29, 30, and 31. The second component of error, that associated with the processing of sampling data by the user, is dealt with by the matrix of variance and covariance values of the independent predictor variables; this sample will be here considered as a simple random sample of the total forest. Stand observations of the predictor variables (X_i's) are used to make predictions of the mean net ten-year stand growth through substitution into the appropriate prediction equations. However, at this point, only an estimate of mean growth is derived, since this random sample is subject to sampling errors the resulting prediction of mean growth also incorporates this error. Errors of the regression equations and errors of the second stage independent sampling of the predictor variables are both considered in the following equation:

$$S\hat{y}^2 = \frac{SE^2_{\text{est}}}{N} + \sum_{j=1}^{p}\sum_{k=1}^{p}\left[\frac{b_j b_k S^x_{jk}}{M} + (\hat{X}_j - \bar{X}_j)(\hat{X}_k - \bar{X}_k)S^b_{jk} - \frac{S^x_{jk}S^b_{jk}}{M}\right] \qquad (2)$$

where: $S\hat{y}^2$ = estimated variance of the predicted mean growth given by the set of M independent obs. of X's.

$SE^2_{\text{est}} \cdot x$ = standard error of estimate from the N set of samples

$$= \sum_{i=1}^{N} (Y_i - \hat{Y}_i)^2/N - p - 1$$

p = number of independent variables in the equation

N = number of observations on which the equations are based (first stage sampling)

M = number of observations in the second stage sampling

b_i = p regression coefficients, derived from N set

S^b_{jk} = estimated variance and covariance matrix of the X_i's of the predictor M set

S^x_{jk} = variance and covariance matrix of the X_j's of the N set, also expressed as $S^2_{y \cdot x} \cdot C_{jk}$

$\hat{X}_j$ = mean of the jth variable in M set

$\bar{X}_j$ = mean of the jth variable in the N set

For a simple linear prediction equation with $p = 1$ and the M set of stand observation of the variable X_j derived from a simple random sample; i.e., subject to

sampling error, the equation expressing the variance of the predicted growth is as follows:

$$S^2_{\hat{y}} = \frac{S^2_{\hat{y}\cdot x}}{N} + \frac{b^2 S^2_x}{M} + \frac{S^2_{\hat{y}\cdot x}}{\Sigma X^2}(\hat{X} - \bar{X})^2 - \frac{S^2_{\hat{y}\cdot x}}{\Sigma X^2}\frac{S^2_x}{M} \qquad (3)$$

where

$$S^2_x = \sum_{i=1}^{M} \frac{(\hat{X} - \bar{X})^2}{(M-1)}$$

$$\bar{X} = \sum_{i=1}^{M} X_i/M$$

$$\Sigma X^2 = \sum_{i=1}^{N} (X_i - \bar{X})^2$$

When two or more independent variables are utilized for the regression prediction of growth, the covariance term S^b_{jk} is easily expressed as the inverse of this matrix by the use of C_{jk} multipliers; since

$$S^b_{jk} = SE^2_{\text{est}} \cdot C_{jk} \text{ and} \qquad (4)$$

$$S^b_{jk} = \frac{SE^2_{\text{est}}}{N \cdot SD_j \cdot SD_k} \cdot (r_{jk})^{-1}$$

then

$$C_{jk} = \frac{(r_{jk})^{-1}}{N \cdot SD_j \cdot SD_k}$$

where:

$(r_{jk})^{-1}$ = the elements of the inverse of the correlation coefficient matrix

Expanding equation (3) to two independent variables and substitution of the value $(S_{\hat{y}\cdot x} \cdot C_{jk})$ for the term S^b_{jk}, we derive the following computational equation for the solution of the variance of the predicted growth:

$$S^2_{\hat{y}} = \frac{SE^2_{\text{est}}}{N} + \frac{1}{M}\cdot(b_1^2 \cdot S_{11} + 2b_1 b_2 \cdot S_{12} + b_2^2 \cdot S_{22}) + SE^2_{\text{est}}\Big((\hat{X}_1 - \bar{X}_1)^2 \cdot C_{11}$$
$$+ 2[(\hat{X}_1 - \bar{X}_1)(\hat{X}_2 - \bar{X}_2)\cdot C_{12}] + (\hat{X}_2 - \bar{X}_2)^2 \cdot C_{22}\Big) - \frac{1}{M}\cdot SE^2_{\text{est}}$$
$$\Big(S_{11}(C_{11} + C_{12}) + S_{12}(C_{11} + 2C_{12} + C_{22}) + S_{22}(C_{12} + C_{22})\Big) \qquad (5)$$

From the M set of stand observations the user determines the mean values of the predictor variables and the covariance terms (S^x_{jk}). The values required from the M set are used in conjunction with the b_j, $\bar{X}_j$, and C_{jk} of the N set of observations. The working equations for the two, three, and five variable equations required for the computation of the predicted growth variance all follow the pattern shown in equation (5), the matrix version of this pattern is shown in appendix F. Examples of the variance of predictions of the basal area and cubic-foot volume growth of the ten samples shown in table 14 have been solved and are presented in an abbreviated form. The basic values required for the solution of the variance equations from the set of ten samples

(*M* set) are shown in the following table, the values from the *N* set are in appendix tables 28 and 29.

VALUES OF THE COVARIANCE MATRIX and MEANS OF THE INDEPENDENT VARIABLES OF THE SAMPLES IN TABLE 14.

Variable	*BA*	*Site*	*Age*	$\hat{X}$
Log basal area...	.0678	.00175	.0274	2.29109
Log site..	.00175	.000869	.00144	2.21093
Log age..	.0274	.00144	.0154	1.45655

Presentation of the variance computation equations with the proper values substituted requires too much space; however, results of the computations are summarized as follows:

1. *BA* growth variance =

$$S_{\hat{y}}^2 = .0001214 + .0006656 + .0000175 - .0000088 = .0007957$$

$$S_{\hat{y}} = \sqrt{.0007957} = .02821$$

Log *BA* growth = 1.955866 ± 0.02868

or in terms of square feet of basal area growth this is 90.335 ± 5.86 square feet per acre per ten years.

2. Cubic-foot volume growth variance =

$$S_{\hat{y}}^2 = .0000553 + .0020542 + .0000219 - .0000061 = .0021253$$

$$S_{\hat{y}} = \sqrt{.00212530} = 0.04610$$

Log cubic-foot volume growth = 3.52884 ± 0.0461

in terms of cubic-foot measure the periodic growth is 3380 ± 358.8 cubic-feet per acre per ten years.

The values of the standard deviations of predicted stand growth given above represent the confidence interval of one standard deviation or approximately 67 per cent. Establishment of confidence interval about the predicted mean growth requires use of the *t* statistic for the chosen level and N-p-1 degrees of freedom.

COMPARISONS TO OTHER REDWOOD STUDIES

Growth studies of young redwood stands are not common; this has been partially due to the overwhelming importance of old-growth stands to the economy of the region and landowners. Preoccupation in harvesting this more economically valuable resource has resulted in a cursory interest in problems relating to utilizing and managing the thousands of acres occupied by less spectacular young stands. Investigations by some landowners have provided some background information of stand growth which is available for comparisons to the current investigations. Unfortunately there are but a few permanent remeasured plots whose records are available for study, most of these unpublished. Some plots which have been examined are summarized in table 15. These growth figures provide a means of assessing the validity of equation predicted net periodic growth. Measured inventory and growth values of these permanent plots have been calculated, during the course of this study, utilizing standards as those for the sample data of the prediction equations. Records of this type provide a valuable indicator of the ability of the equations to approximate some measured growth performances. Some of the discrepancies between actual and predicted growth result

Table 15. Predicted Growth vs. Measured Growth of 20 Permanent Remeasured Young-Growth Redwood Plots (Annual Basis), Board-Foot Volumes by International 1/4 Inch Rule

Plot	Area	Growth period	Initial age	Initial basal area	Site	BA Growth		Vol. Gr.		> 10.5″		> 15.5″	
						Act.	Pred.	Act.	Pred.	Act.	Pred.	Act.	Pred.
	acres	*years*	*years*	*sq ft*	*index*	*sq ft*		*cu ft*		*bd ft*		*bd ft*	
1.....	.2	5	34	325	164	11.7	10.3	571.0	450.5	3,431.1	2,712.0	2,769.4	2,879.0
2.....	.2	5	17	112	155	12.8	10.8	285.6	294.4	1,200.2	1,368.0	770.3	1,435.0
3.....	.2	5	19	97	147	9.3	8.9	182.3	237.9	988.9	1,163.0	891.0	1,269.0
4.....	.2	5	37	372	167	6.7	10.3	513.9	480.1	3,607.2	2,996.0	3,899.8	3,192.0
5.....	.2	5	33	280	170	9.1	9.7	411.0	419.0	2,708.1	2,575.0	2,639.3	2,750.0
6.....	.2	5	33	127	179	7.1	6.1	299.4	238.3	1,966.9	1,574.0	1,940.4	1,915.0
7.....	.2	5	35	237	143	7.6	8.3	309.3	316.3	2,005.1	1,814.0	2,064.2	1,987.0
8.....	.2	5	34	303	164	10.6	9.9	509.0	427.1	2,630.3	2,583.0	3,465.4	2,754.0
9.....	.2	5	35	368	174	9.1	10.7	493.5	506.6	3,495.9	3,178.0	3,433.7	3,366.0
10.....	.2	5	20	66	166	9.1	6.8	242.0	186.8	1,455.0	1,071.0	1,596.9	1,250.0
11.....	1.0	10	65	597	181	7.4	7.9	507.1	521.5	3,913.1	3,894.0	4,101.7	4,304.0
12.....	1.0	10	75	671	180	6.8	7.3	411.1	520.2	3,361.2	3,853.0	3,629.0	4,300.0
13.....	1.0	10	85	738	180	6.2	6.8	583.2	517.5	4,644.4	3,750.0	4,814.2	4,215.0
14.....	1.0	10	95	800	182	2.4	6.4	360.4	518.6	3,075.0	3,667.0	3,340.0	4,140.0
15.....	.5	10	45	254	168	5.5	6.8	295.3	323.6	2,100.6	2,203.0	2,116.7	2,523.0
16.....	.5	10	55	310	174	4.2	6.3	267.3	341.2	2,049.5	2,515.0	1,983.7	2,905.0
17.....	.5	10	45	282	174	6.1	7.2	397.0	369.4	1,788.8	2,491.0	2,005.8	2,802.0
18.....	.5	10	55	343	172	6.6	6.6	344.8	358.4	2,898.7	2,662.0	2,998.5	3,015.0
19.....	.5	10	45	254	165	5.5	6.8	238.3	319.6	1,510.4	2,151.0	2,061.5	2,463.0
20.....	.5	10	55	308	165	6.2	6.3	423.9	327.4	3,149.8	2,322.0	3,015.2	2,680.0
Total growth.....						150.0	160.2	7,645.4	7,673.2	52,980.2	50,542.0	53,540.7	56,144.0
Average growth.....						7.5	8.01	382.3	383.6	2,649.0	2,527.1	2,677.0	2,807.2

from cutting or excessive windthrow, as the result of cutting, during the growth period. Despite these individual plot differences, the comparisons give an impression that, on the average, predicted growth is a reasonably accurate approximation of measured growth.

The most obvious comparisons to be made of these predicted growth rates are to results of stand growth of the yield tables developed from this data (Lindquist and Palley, 1963). Differences in periodic growth rates are due largely to the processes used in each study for evolving the periodic stand growth. Factors relating to methods of construction and application need to be considered when viewing these two approaches to estimation of future periodic stand growth. Prediction equations portray growth as a function of three basic variables: age, site, and basal area; all appear vital to explanation of stand periodic increment. Opposed to this growth, as depicted in yield tables, is assumed to be an average across a wide range of possible basal area levels for each age and site class. Inclusion of initial stand basal area in the equation format more closely defines the stands for which predictions of future growth are desired. Curves of growth developed by repeated solution of the prediction equations do not define growth trends of stands over a period of time as is the case of the yield tables. Yield table periodic growth is the difference between stand yields for consecutive ten-year ages, and is the result of work with terminal inventory values of the stands. Present growth equation evaluation, stipulated for 10-year periods, studies actual measured periodic growth of each stand observation; then expresses the regression equation of this growth and associated initial inventory values of age, site, and basal area.

An advantage of the growth predictions over those from yield tables is the allowance for different levels of initial basal area. The stand that currently occupies a site is the result of conditions existing at the time of stand regeneration; it is not entirely safe to assume that basal area levels shown in yield tables will be valid for a particular stand. It is this comparative inflexibility of yield tables which, when dealing with stands stocked materially different than shown in yield tables, has caused the derivation of methods to adjust yields and subsequent growth. Growth tables in this prediction study show relatively uniform growth rates over a wide range of basal area levels; this is especially true for stands in excess of 50 years of age.

Long range stand development trends, as shown in yield tables, have a tendency to reduce climatic influences to an average of conditions that have occurred over the entire range of ages represented by the stand observations. The short-term equations, based on a fixed period of calendar years, removes some variation of stand growth caused by these climatic fluctuations by measuring periodic growth of stands of all ages under relatively uniform climatic influences. Complete elimination of climatic factors is not possible since the sample data represents a geographic distribution where climate is not entirely similar for the growth period measures. A further element to consider when dealing with short-term equations is the assumption that the climate in subsequent 10-year periods will be similar to that of the base measured growth period. The degree of

effect on stand growth of markedly different climatic regimes during the 10-year predicted growth period cannot be evaluated; however, it is well to consider that there will be some stand reaction.

Yield tables prepared by Bruce (1923) provided the basis for much of the investigations carried on in this timber type since their publication. Comparisons of growth in the present study with Bruce's normal yields and growth presents problems brought about by definition of the stands which functioned as the basis for the studies. The concept of stand normality, as defined by Bruce, restricted selection of stands which could be accepted as legitimate samples; this restriction to fully stocked stands did not function in the selection of sample units in the present study. Additional differences in the minimum tree size, site index curves, and log rules for evaluating the yield and growth data all contribute to the making of definite comparisons of the two studies tenuous. Generalizations of trends of stand growth are possible and provide an indication that through a portion of the Bruce results, there is a degree of agreement of growth rates in the younger ages of the stands. The most divergent feature of the Bruce studies is that basal area growth has nearly reached zero between 55 and 60 years of age. This reduction in the growth of the horizontal element of the stands has to affect volume growth, since increase in height would be the only factor bearing on the volume growth of the stands. Reduction in volume growth is noted in both the cubic- and board-foot growth of the three site classes, periodic volume growth is cut back sharply between 50 and 60 years. The present studies indicate a decreasing growth with increasing age, but that the level of increment does not reduce to levels indicated in the Bruce studies. Direct comparisons between growth rates are not possible without the postulation of basal area values for the ages and sites that are to be compared. Using initial stand values of Bruce's tables as basis for solving the prediction equations indicates that prediction equations will develop larger basal area and volume growth values than shown in the Bruce yield tables.

Two references of the stand growth of young redwood, Carr (1958) and Hallin (1934), also show values that in general support the growth rates of the prediction equations. Carr's investigations involved three one-acre plots, two thinned and one unthinned, established in stands on the flats of the Gualala River, a high site (approximately 190, by the included site index curves). Periodic annual growth, based on a four-year period, of trees larger than 11.00 inches at breast height using the Spaulding Rule was 3,861, 3,260, and 2,975 board-feet per acre per year. Conversion of the predicted International $\frac{1}{4}''$ rule of the equations for the stand larger than 10.5 inches (average stand diameter is 22 inches, figure 7, or 96 per cent) gives predicted Spaulding values of 3,460, 3,400, and 2,795 board-feet per acre per year, which are reasonably well related to the actual measured growth. The reference by Hallin to a stand approximately 260 years old when cut indicated that the stand during the first 100 to 150 years had a mean annual increment of 3,500 to 4,000 board-feet per acre. A mean annual increment of this magnitude means that there were undoubtedly periods which equaled or excelled rates

of periodic growth shown by the board-foot equations of this study.

The type of prediction that an owner of this timber type requires in any decision should affect his choice of either the prediction formulas or the yield tables. For information required of extended periods of time (20 years or more) the logical source is the yield tables, which are based on long-range averages of the inventory data over the period of years covered by the tables. Prediction of growth for periods up to 20 years may be best made with the equations presented in this study, although projection beyond the 10-year period for a stand is not highly recommended. Despite differences that exist between growth values for similar stands using both processes of prediction, it should be kept in mind that they have been developed via different routes and in fact are not used for the same type of required information. Rates of growth of these stands as estimated by the prediction equations are generally larger than those shown for comparable age and site classes of either the 1923 or 1963 yield tables; however, the equations develop growth results which agree more closely with the actual productivity of the few remeasured plots that are available for study.

SUMMARY AND DISCUSSION

This study has brought out several items of interest. Most important are those that affect the applicability of the growth tables to actual management procedures. Other items which have been broached are the need for further investigations of stand information before a clear picture of the future management of young-growth redwood can emerge.

When applying the growth tables of this bulletin to the thousands of acres of unmanaged stands, care must be given to their proper implementation as defined by this study. Extensive field testing of these tables must wait the passage of time to determine how well the equations predict net stand growth. It is expected that over the ensuing years as more of the young growth is harvested and put under a form of utilization, these tables may not be entirely applicable. At some time in the future the tables will have to be reassessed to determine the changes in growth patterns brought about by the institution of management in these natural stands. At the present time, there is little land in managed condition with which to compare the growth tables.

The growth equations have an advantage of using an actual measure of the stand basal area to which they are to be applied. The influence of this stand stocking measure on growth is a vital factor in the expected periodic volume growth of these stands. Growth as predicted by these equations appears to be higher than that shown by comparable yield table figures. It is well to consider that the growth values are not adjusted for losses that occur through harvesting breakage or culling, but represent net stand growth of all trees above the set minimum diameter limits. In applying the growth predictions, it is not desirable to extend them much beyond the 10-year period because rates cannot be sustained for longer periods.

Growth experience for the few re-

measured plots that are available gives a degree of credence to the values reported in the growth tables. Although individual stand differences are expected to occur, the trends shown by remeasurements seem to bear out the applicability of these growth tables to actual growth situations found in this timber type.

Management criteria necessary in long-range planning of a forest ownership are complex and go far beyond questions of stand growth. What product is desired, and economically feasible under a set of production conditions, depends on market demands and on whether these stands have the capacity to develop this type product. Economic returns to owners involve an analysis of the expected value of the added yield as a function of the money invested in the stands over a period of time. Other areas dealing with forest regulation such as silvicultural treatment and response, increasing risk with age, and harvesting and manufacturing equipment, all have a bearing on management's final decision how it will dispose of a forest property. Our growth tables do not attempt to show the owners of young-growth redwood the most appropriate solution for all considerations, but rather provide an additional tool to facilitate planning. Broad management alternatives can be implied from our study; for example, short rotations for small products such as pulp roundwood seem possible because yields are relatively large at early ages. These same stands also have a capacity for good growth rates up to ages required to develop saw logs for lumber production. The possibility of integrated product operations based on thinning products is feasible if markets develop for this size logs and cost of thinnings are not prohibitive. It is not possible, on the basis of this study of natural stands, to make definite statements about the expected growth of stands which undergo partial cutting; the effects of release by thinning are not available. It seems reasonable to theorize that stand improvement operations should result in improved quality and possibly increasing growth on trees left for eventual saw log production, and in addition recover volume normally lost through natural causes.

Experience has shown that this species regenerates well in heavily cutover land; the majority of the sampled stands in this study were regenerated during the years of clear cutting. Serious questions were left unanswered because the quality of stand restocking leaves something to be desired in terms of the spatial distribution of redwood. The distinctive stem distribution generally found in these young-growth stands is the result of three factors: the ability of this species to sprout profusely about the stump, the initial distribution of the old-growth stems, and the lack of effective redwood seedling regeneration. Show and Stuart (1932) estimated that sprouting of redwood would at best account for approximately 35 per cent of full stocking, but that a few seed trees of the whitewood species would develop mixed stands that were 80 per cent or better of full stocking. The concept of full stocking, as used, appears to be tenuous when it is not uncommon to find nearly pure redwood stands which have attained large basal areas at relatively young ages. What is important is that this redwood basal area is often concentrated in only a few clumps per acre, and that over large distances the stands do not have stems of any species.

The unequitable utilization of the land area by stems is perhaps the basis for the assumption that redwood will not adequately regenerate the land if the restocking is solely by sprouts. Establishment of the second crop of trees on the root systems of the older stands is perhaps misleading; these clumps may in fact completely utilize the land space since the old trees had developed much larger roots than would be necessary for the smaller trees now in place.

Looking to the future and the results of harvesting current stands, much is yet in the area of conjecture and poses serious problems in planning reasonable management of young stands of this species. Fritz (1959) has posed and discussed questions that owners should consider in implementation of any form of stand management. Clear cutting, if carried out on these stands, may result in essentially a pattern similar to the stem distribution with which we now must contend—heavy localized sprouts about the stumps. Partial cutting for thinning operations, either from below or above, to improve growth for quality and recover for sale volume lost through mortality, may not provide conditions for either an adequate sprout or seedling regeneration until the final stand is removed. In effect, this causes a return to an even-aged condition but delays stand regeneration by redwood until the stand is completely removed. These questions must be considered and resolved by further experience with the management of this timber type, before realistic guidelines can be drawn which pertain to management of this timber type.

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APPENDIX A

Stand inventory and growth computations

Evolving estimates of stand growth when the procedure involves temporary sample units requires that tree measurements taken be amenable to reconstruction of stands as they were a fixed number of years prior to sample establishment. By establishing stand samples and measuring stems which constitute a defined stand, it is assumed that growth of individual stems defines stand growth. Moving from the study of the individual stems to the stand, develops defining characteristics of the stand which are available for manipulation and study. It is the procedural derivation of stand inventories and periodic growth from the stem measurements that this section of the study attempts to clarify.

Use of a point sampling technique to identify stems comprising a stand sample results in a material change in the manner by which the inventory values are calculated. In reliance on temporary stand samples to assess growth, there is a requirement for increment cores; these are time consuming—an important consideration in the establishment of samples. This time element is greatly reduced using a point sample, which can restrain the number of stems included, through selection of the basal area factor. There is not an areal basis for the stems of a point sample so that recovery of the stand characteristics on a per acre basis requires an adjustment in calculation procedures and conceptual understanding on the part of those who may wish to apply this form of growth measurement. The reader is directed to articles related to this topic as treated by Grosenbaugh (1952, 1955, 1958). For the present discussion, theoretical aspects of the technique are not treated but are mentioned only in their application and adaption to the calculation procedures in this growth study.

Several broad objectives which the authors felt were required in a comprehensive study of stand growth and would guide calculation of the sample data are summarized as follows:

1. Per-acre inventory, net- and gross-growth should be calculated for numbers of trees, basal area, volumes, and average diameter.
2. Each of three inventory dates would be considered: the present (1958), five years ago (1953), and 10 years ago (1948); this results in developing growth for a five- and 10-year period.
3. Species would be identified and volumes calculated separately for each of three groups: redwood; whitewood (all conifers other than redwood); and hardwood.

4. For each inventory date and species the inventory and growth results would be calculated for all trees larger than 4.5 inches DBH, the cubic-foot stand; and for trees larger than 10.5 and 15.5 inches DBH, the board-foot stands.
5. Calculation as separate features, the ingrowth and mortality during the five- and 10-year periods of each stand characteristic for all minimum diameter limits.

The task of the reduction of data in the sample units established to a per-acre basis, including all the basic objectives, forced the evaluation of the possibility of developing a linear computational program for a high-speed electronic computer. Early in 1959 design of a program to incorporate the required calculations was undertaken and a workable computational procedure was fitted to the IBM 650. Preliminary computations of samples measured during 1958 were done on this machine; however, the size of the computer required several restrictions in computational procedures in order to stay within the limited memory capacity of the machine. Subsequent access to the IBM 704 and 7090 at the University of California Computer Center allowed redesign of the program to effect more complete and better integrated computational procedures.

The flow of data under control of the computing program begins with the reading of a plot card; this card states the basal area factor used in the selection of trees for the sample and also the number of trees which will follow.[4] A separate card for each tree of the sample unit then follows; these tree cards are read one at a time and entirely completing the classification into the various categories that have been set up. Developing the computed inventory values of stands, based on the stems included in each sample, is predicated on this detailed classification system which sorts and accumulates two values related to each stem. These categories depend on species, inventory date, the minimum diameter limits of the volume units, and the ingrowth and mortality designations. The basic values calculated and accumulated for each stem are the reciprocal of the square of the terminal diameter, and the ratio of the inventory diameter to the terminal inventory diameter. For each classification, an accumulator of the values required in the computation of the per acre characteristics is set up which functions in the following form:

$$A(M,I) = A(M,I) + 1/(D_t)^2 \qquad (6)$$

$$B(M,I) = B(M,I) + (D_i)^2/(D_t)^2 \qquad (7)$$

where:

A and B = names of stored memory arrays.
M = species code; 1, rwd; 2, whwd; 3, hdwd.
I = an index referring to the combined pattern of inventory dates and volume units; i.e., odd numbers refer to cubic-feet, and even numbers to the board-foot inventories.
D_i = Stem diameter of the inventory being computed.
D_t = Stem diameter at time of inventory on which all sampling probabilities depend.

The summing of values for each tree in each category into which it falls begins the process of stand calculations

[4] For a further discussion of computational techniques, see Palley (1963).

and submerges the identity of individual trees comprising a sample. These summed values plus local volume line coefficients discussed later provide basis for the subsequent calculation of all inventory and growth figures.

Within the classification portion of the program, there occurs an identification of what are termed height sample trees. These trees, generally dominants, were selected for measurement of total height. Each coniferous group represented in a stand sample required that stand volume be computed; this necessitated construction of a local volume table for that species group in each of the inventories that this species occurred. Because of the differences in form of redwood and the whitewood species, it was felt that they could not be combined into a single local volume equation; they are considered separately. The sequential steps in deriving stand volumes for each of the categories begins with the calculation of the local volume lines from those trees whose total height was inserted on the data card, generally three to five redwood. The first step in the process was to calculate stem volumes (cubic- and board-foot) of each height sample tree; this is done by application of stem diameter and height to equations which express standard volume tables; summarized in table 24. The program then calculated and summarized computational elements required for the solution of each local volume line as a simple regression equation of stem volume over diameter squared. These elements are: sum of diameter squared, D_i^2; sum of volume, V_i; sum of cross-products, D_i^2; and the sum of the square of diameter squared, $(D_i^2)^2$. Computation of the local volume lines was done in the second portion of the program after completion of the classification of all stems in a sample. Because of the restriction of minimum diameters for trees in each volume unit, it was necessary to develop local volume lines with a formula slightly different than the normal least square formula. Also computation of a regression line with only one to five points may result in local volume lines that were not realistic, i.e., a least-square line may be a line that intercepts the volume axis at a level which indicates that trees of all diameters have the same volume. To forestall this, the cubic-foot equation was restricted to interception of the diameter squared axis at a point of a four-inch tree, i.e., a tree of this size has zero volume. A similar restraint forced the board-foot volume lines to intercept the diameter axis at the 10-inch point. As a result of the restraint on the regression line, there is a negative value for the intercept of the volume line with the volume axis. The formulas for computing the slope and intercept coefficient of each local volume line are of the form:

$$b = \frac{\Sigma XY - K\Sigma Y}{NK^2 - 2K\Sigma X + \Sigma X^2} \qquad (8)$$

$$a = -bK \qquad (9)$$

where:

X = diameter squared
Y = volume
N = number of trees
K = point of intercept of volume line and X-axis; 16 for cubic-feet, and 100 for board-feet.

These formulas were applied to height sample trees representing each coniferous group of three inventory dates; calculated regression coefficients were stored for use later with the accumu-

lated values for computation of stand volumes. For hardwoods these local volume lines were not developed for each sample, since heights were not measured. Cubic- and board-foot local volume tables developed for tanoak (*Lithocarpus densiflorus*) and red alder (*Alnus rubra*) in this region by the United States Forest Service were combined and expressed as coefficients of simple regression equations (table 24). Regardless of sample unit or inventory these coefficients were used for calculation of total stand volume contributed by hardwoods.

Since local volume lines were to be computed for each of the softwood species groups for both volume units in each of three inventories, the height sample trees had to be reconstructed to recover initial height and diameter. The reconstruction of the initial diameters has been discussed previously. To ascertain part height of height sample trees, site index curves were used; for young-growth redwood (Lindquist and Palley, 1961) and Douglas-fir (McArdle *et al.*, 1949). Stem height and age as recorded in the field established the site index of each tree, then by following the site curve back ten years the part height of each stem was determined. On the data card, height as measured and height ten years ago were inserted, height for 1953 was computed by the program as half the difference between the 1958 and 1948 measurements.

Upon completion of the classification and accumulation phase, when all stems of a sample have been read into the program, control passes to the section of the program which develops the per acre inventory and growth of the four stand characteristics. Accumulation of computational elements is done by species, volume unit, and inventory date; the calculation of stand values is accomplished by repeated solution of four basic equations:

$$\text{No. trees per acre} = A(M,I) \cdot BAF/.005454 \tag{10}$$

$$\text{Basal area per acre} = B(M,I) \cdot BAF \tag{11}$$

$$\text{Volume per acre} = BAF/.005454\ (a \cdot A(M,I) + b \cdot B(M,I)) \tag{12}$$

$$\text{Av. dia. per acre} = \sqrt{\frac{\text{basal area per acre}}{\text{Trees per acre}} \cdot \frac{1}{.005454}} \tag{13}$$

This generates for each category of M (species) and I (inventory and volume unit) the basic descriptive statistics of each category in terms of the four stand characteristics. The constant term in these equations, .005454, converts measurements of diameter in inches to units of square feet of basal area.

Number of trees per acre in the initial inventories despite stem diameter growth over the period remains constant as a result of using the diameter measurements of the terminal inventory. This stand value will be different in initial inventories only when trees have exceeded the minimum diameter limits or died during the growth period. Thus a tree present and alive in all inventories will contribute an equal weight to calculation of stems per acre. The evaluation of basal area per acre is accomplished by the accumulated sum of the ratio of diameters. For the terminal inventory, 1958, the

ratio of each tree is one (1.0) so that in the final analysis computation of terminal inventory basal area per acre is simply the count of tree accumulated times the basal area factor. The ratio for each tree in the initial inventories is always somewhat less than one if there has been any radial growth during the period.

Total values of the inventories for the four stand characteristics are arrived at by summing the values for the three species. Once these statistics are available the next step is the computation of net growth, defined as the difference between the terminal and initial inventory of each stand characteristic. This develops for each of four stand statistics five- and 10-year net growth of the stands defined by the minimum diameter limits. To evaluate the gross stand growth, the categories of ingrowth and mortality must be considered. It is necessary to clarify the definition and handling of these categories in evaluation of the computational procedures.

Ingrowth as used in this project refers to those trees which, during either the five- or 10-year periods, attained a size larger than the minimum diameters of 4.5, 10.5, and 15.5 inches DBH. In collecting the data, trees less less than 4.5 inches DBH were not recorded, so all living trees of a sample were eligible for the 1958 cubic-foot inventory. Radial cores indicated some trees to be less than this size for either the 1953 or 1948 inventory computations; these trees were accumulated as ingrowth. If trees attained minimum size in the period 1958–1953, their contribution to basal area, volume, etc., was accounted for in both the five- and 10-year periods; if during 1953–1948, only in the 10-year period. Ingrowth of the board-foot stand followed the same pattern; however, the minimum diameter limits are 10.5 and 15.5 inches at breast height. Computation of the four stand characteristics for each ingrowth category has the same pattern as for inventory values; the exception to this is the use of the 1958 volume line coefficients for computation of the ingrowth volumes, regardless of the growth period in which trees surpassed the minimum diameters.

Trees dead less than 10-years required inclusion of their contribution to the inventory in which they were last alive; also the contribution of these trees was added to the five- and 10-year mortality values, depending on the length of time dead. Diameter limits of the volume units were used in this calculation so that a dead tree larger than the minimum diameter limit was included in all categories of mortality. Calculation of the characteristics of mortality was done in the same manner as the inventory and ingrowth, with the exception that no cores were removed from dead stem; this meant that the diameter as measured at the time of sample establishment is used for all calculations. For computation of mortality volume, the volume line coefficients of the 1953 inventory were used for trees which died during the 1958 to 1953 period and the 1948 inventory coefficients for the stem that died between 1953 and 1948.

Accounting of the ingrowth and mortality of the five- and 10-year growth periods permits the adjustment of the net growth to gross growth; this is done by adding the mortality and subtracting the ingrowth to the values of five- and 10-year net growth. This is a simple process for stand growth values of frequency, basal area, and volumes;

however, for average diameter net growth, the mortality and ingrowth average are not amenable to this direct adjustment. A special set of calculations which returns to the adjusted initial and terminal basal area and stems per acre is required to evolve this gross average diameter growth.

APPENDIX B

Sample distributions and descriptive information

Processing sample data and further refining criteria for defining sample units has brought about rejection of some sample units of the total set established. Of the 172 original stand samples, nine were removed from consideration in this study of 10-year growth; rejections were as follows:

1. Residual old-growth stems	4
2. Less than 20 per cent redwood	2
3. Missing data	3
Total	9

The first category results from included old-growth stems in these samples being an integral part of the young-growth stand. In many stand samples, where old-growth was encountered, the sample point was so located as to exclude these old extremely large trees. It was not deemed reasonable, in a study of young stands, to include residual stems which have large volumes and growth patterns different than younger vigorous stems. In these four sample units, the old-growth stems were so situated that it was not reasonable to remove the old trees; it was felt that the entire sample should not be considered. Samples rejected for use in preparation of yield tables, derived from this same sample data, because of abnormal basal area levels or ages over 100 years, were retained in this study which treats age and basal area as continuous variables. These samples serve to extend the range of the prediction equations by providing growth values for these over- or understocked stands.

Tables 16, 17, and 18 indicate distributions of the sample units on the basis of the initial reconstructed inventories; thus stand age and basal area refer to the stand as it was 10 years prior to sample measurement. Site index is based on the average breast-high age and total height of dominant redwood as measured at the time of sample establishment; site index is assumed to have remained constant over the preceding 10-year growth period. Other distributions of the stanp samples, in regard to some of the environmental conditions noted in the sampling process, are shown in tables

Table 16. Frequency Distribution of Sample Points by Initial Breast-High Age Class and Site Index Class

Age at B. H.	Site index						Total
	Over 200	181–200	161–180	141–160	121–140	101–120	
years	*sample units*						
0–10	..	1	3	1	..	..	5
11–20	3	5	6	2	2	..	18
21–30	7	12	9	7	1	..	36
31–40	5	8	15	3	3	1	35
41–50	2	10	11	6	2	..	31
51–60	2	5	4	5	2	1	19
61–70	2	1	4	..	1	..	8
71–80	1	1	1	..	..	..	3
81–90	1	1	1	1	..	..	4
91–100	..	..	..	..	..	..	..
>100	..	3	1	..	..	..	4
Total	23	47	55	25	11	2	163

Table 17. Frequency Distribution of Sample Points by Average Initial Breast-High Age Class, and Initial Basal Area Per Acre

Age at B.H.	Basal area per acre (sq ft)										
	0–100	101–200	201–300	301–400	401–500	501–600	601–700	701–800	801–900	>901	Total
years	*sample units*										
0–10	5	..	..	..	..	..	..	..	..	..	5
11–20	10	4	3	1	..	..	..	..	..	..	18
21–30	2	13	15	4	2	..	..	..	..	..	36
31–40	3	5	9	11	4	2	1	..	..	..	35
41–50	..	3	8	12	4	3	..	1	..	..	31
51–60	..	3	3	8	1	3	1	..	..	..	19
61–70	..	..	..	4	3	..	1	..	..	..	8
71–80	..	..	..	..	..	1	1	1	..	..	3
81–90	..	..	..	1	..	1	1	..	..	1	4
91–100	..	..	..	..	..	..	..	..	..	..	..
>100	..	..	..	..	..	..	1	..	1	2	4
Total	20	28	38	41	14	10	6	2	1	3	163

Table 18. Frequency Distribution of Sample Points by Initial Basal Area Per Acre and Site Index Class

Basal area	Site index						
	Over 200	181–200	161–180	141–160	121–140	101–120	Total
sq ft	*sample units*						
0–100	..	3	9	4	3	1	20
101–200	1	7	10	6	4	..	28
201–300	6	12	9	8	2	1	38
301–400	4	12	17	6	2	..	41
401–500	5	5	4	..	..	..	14
501–600	3	3	3	1	..	..	10
601–700	3	2	1	..	..	..	6
701–800	1	..	1	..	..	..	2
801–900	..	1	..	..	..	..	1
> 900	..	2	1	..	..	..	3
Total	23	47	55	25	11	2	163

Table 20. Frequency Distribution of Sample Units by Slope Class and Site Index

Site index	Slope Class (per cent)			Total
	0–10	11–30	31	
	sample units			
> 201	19	2	2	23
200–181	17	20	10	47
180–161	15	16	24	55
160–141	5	7	13	25
140–121	..	4	7	11
120–100	..	1	1	2
Total	56	50	57	163

Table 19. Frequency Distribution of Sample Units by Aspect (Cardinal Direction) and Site Index

Site index	Aspect					Total
	North	East	South	West	Level	
	sample units					
> 201	3	..	1	2	17	23
200–181	6	4	13	10	14	47
180–161	7	8	15	14	11	55
160–141	4	4	8	4	5	25
140–121	2	1	6	2	..	11
120–100	1	..	..	1	..	2
Total	23	17	43	33	47	163

Table 21. Frequency Distribution of Sample Units by Parent Material Group and Site Index

Site index	Soil group			Total
	200*	800†	900‡	
	sample units			
> 201	14	3	6	23
200–181	9	19	19	47
180–161	3	29	23	55
160–141	1	15	9	25
140–121	..	10	1	11
120–100	..	1	1	2
Total	27	77	59	163

* Alluvial
† Consolidated sandstone
‡ Weakly consolidated material

19, 20, and 21. Relating the aspect, slope class, and parent material class to site index provides a view of the range of conditions encountered in the development of the sample data. It should be considered that the system of sampling was purposive, not random; thus the sample distributions are not necessarily indicative of the distributions occurring in the total population of the young-growth stands. These distributions show general trends of sample occurrence within the site index classes established for this study.

Consideration of stand growth for different minimum diameter segments of the sampled stands has resulted in a different number of samples for the various growth components. One sample was removed from the study of trees over 10.5 inches DBH, and five from the study of trees over 15.5 inches DBH; no trees in these samples exceeded the minimum diameters. Thus independent variables which serve as predictors in the analysis have some differences. Average values for the prediction equation variables, age, site index, and basal area (trees over 4.5 inches) are shown in table 22. This table also indicates the average values of other independent and dependent (growth) variables that define the stand sampled. Some values are not presented, i.e., cubic-foot volume growth for trees over the two larger diameter limits, since these values were not computed for all the diameter segments considered in this study.

The simple correlation coefficients between independent and dependent variables investigated in this work are summarized in table 23. These values

Table 22. Summary of the Reconstructed Initial Inventory Mean Values of Pertinent Stand Characteristics and Stand Growth Used in the Prediction Equation Development

Variable	Values for trees larger than these minimum diameters					
	4.5″		10.5″		15.5″	
	Mean	S.D.	Mean	S.D.	Mean	S.D.
Stand age (yrs)	40.4	±22.9	40.6	±22.8	41.4	±22.6
Site index	175.1	±23.1	175.0	±23.1	175.4	±23.3
Basal area (sq ft)	306.7	±190.8	273.0	±196.0	218.2	±200.0
Frequency (trees)	257.6	±173.5	140.0	±76.9	75.4	±49.9
Cubic-foot volume	9,922.3	±10,326.0	.....	.....	.....	.....
Board-foot volume	.....	.....	55,500.0	±73,200.0	50,773.0	±73,901.0
Av. diameter (inch)	16.0	±6.92	18.2	±6.2	20.6	±6.8
Basal area growth (sq ft)	94.5	76.6	.....	.....	.....	.....
Cubic-foot growth (cu ft)	4,139.0	±1,877.0	.....	.....	.....	.....
Number of samples	163		162		158	

Table 23. Partial List of Simple Correlation Coefficients of Independent and Dependent (Growth) Variables Considered in This Study. Independent Variables Refer to the Reconstructed Initial Inventory Values

Variable	Net basal area growth (>4.5″) (163 plots)	Net cubic-foot growth (>4.5″) (163 plots)	Net board ft. growth (>10.5″) (162 plots)	Net board ft. growth (>15.5″) (158 plots)
Age	−.400	.050	.275	.301
Age²	−.308	.002	.297	.177
1/age	.294	−.184	−.403	−.415
Log of Age	−.411	.121	.383	.393
Site Index	.264	.625	.673	.632
Site²	.283	.641	.295	.640
Log Site	.242	.601	.659	.617
Age and Site	−.334	.157	.380	.401
Number of trees per acre W.S.*	.585	.554	.317	.215
Log of trees per acre W.S.	.573	.570	.381	.258
Diameter W.S.	−.418	.033	.297	.350
Log of Diameter W.S.	−.444	.063	.386	.417
Number of trees per acre S.S.†	.184	.550	.657	.704
Log of trees per acre S.S.	−.146	−.383	.534	.566
Diameter S.S.	−.350	.123	.340	.418
Log of Diameter S.S.	−.127	−.275	.348	.392
Height	−.263	.336	.593	.609
Log of Height	−.223	−.607	.650	.651
Basal Area W.S.	−.033	.513	.689	.685
Basal Area² W.S.	−.091	.348	.495	.507
Log of Basal Area W.S.	.045	.599	.776	.744
Basal Area × Age W.S.	−.224	.176	.741	.651
Basal Area × Site W.S.	.009	.567	.602	.602
Basal Area S.S.	−.161	.410	.648	.593
Log Basal Area S.S.	−.183	.339	.587	.577
Cubic Foot Volume W.S.	−.172	.339	.529	.542
Board Foot Volume S.S.	−.230	.249	.448	.438

* W.S. Refers to all trees over 4.5″ DBH.
† S.S. Refers to all trees over 10.5″ or 15.5″ DBH for the two board foot growth values.

have been frequently referenced in the main body of the text as indicative of the relationship between stand inventory values and periodic growth. These values, sample correlation coefficients $= r$, are computed as follows:

$$r = \Sigma xy / \sqrt{(\Sigma x^2) \cdot (\Sigma y^2)} \qquad (14)$$

where:

$x = X_i - \bar{X}$ independent term

$y = Y_i - \bar{Y}$ dependent term.

This statistic varies between $+1$ and -1; the higher the value of r, regardless of sign, the closer the association between the variables. A value of $r = 1$ means perfect correlation, while a value of 0 means no correlation at all between the elements considered. Presentation of this information provides some insight for users into the relative value of the independent stand and site variables considered in the derivation of the growth prediction equations.

APPENDIX C

Mortality considerations

Attrition of stems from stands follows a pattern not easily understood and quantified. We know that regardless of the number of trees that become established the number remaining will decrease over time. Dense stands have heavy losses early in their history—

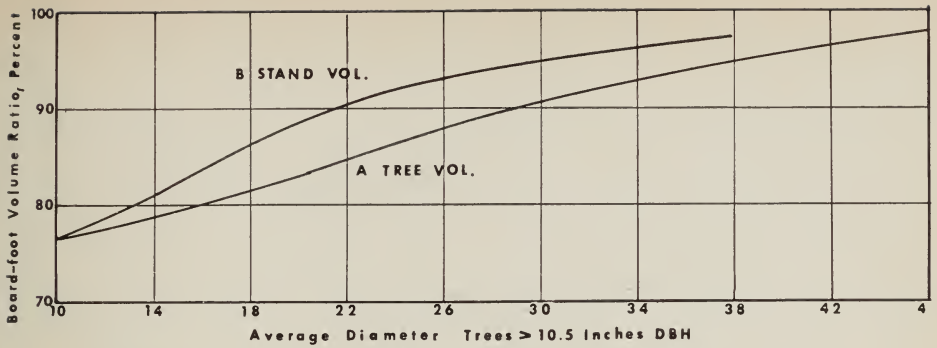
mostly small trees; less dense stands have a smaller early loss; losses continue but at a reduced rate. Eventually, as stands reach the mature ages, numbers of trees moving toward harvest are approximately the same. Stand studies have shown that redwood is very tolerant of competition and that extremely suppressed trees will survive for many years. Mortality figures of sampling the 163 stands used in this study show 36 trees dead less than 10 years on 27 sample units, or in approximately 17 per cent of the total number of plots mortality trees were recorded. Reasons for a low incidence of dead trees are not attributed solely to the survival strength of the species; it is partially the effect of the limitation to trees larger than 4.5 inches in diameter and sampling with an angle gauge. Death of most redwood results from suppression; few pathogenic organisms affect this species, and with sampling restricted to only trees over 4.5 inches the chances of selecting small suppressed stems are less than of the larger diameters. Dead trees may have in fact existed in many stands sampled, but their small size and location relative to the sample point precluded their selection in a sample. The effect of each tree included in samples on the number of trees per acre is proportional to the reciprocal of the square of the stem diameter; the smaller the tree the larger the contribution to stems per acre. Loss of trees per acre due to the death of a single tree may be very large if the diameter is small and the basal area factor for selecting stems large.

This topic of stand mortality is not easily studied from data collected in this study of short-term growth. Possibly the most useful information relative to this topic is inferred from tables of stem frequency in the recent yield tables of this species (Lindquist and Palley, 1963). In these tables are portrayed changes in the numbers of trees above given diameter limits so that the overall situation of tree death in these stands by age and site index is not entirely presented. Except for the lowest site classes numbers of trees per acre over 4.5 inches decrease with increasing age (table 5 of the yield tables); this loss of stems undoubtedly takes place in smaller diameter classes so that loss in terms of volume is of no great consequence. Investigations of patterns of stem loss in available permanent remeasured plots indicate that little information of a definite character could be extracted. Mortality in terms of number of stems, volume, and basal area was very poorly correlated with any of the initial stand characteristics. Dearth of information regarding the loss of stems from these young-growth redwood stands does not allow possibility of a detailed investigation of this topic. It is not the intention to ignore or downgrade importance of stem losses in the study of growth, but to point out that in light of present information little can presently be done with either the permanent remeasured or temporary plots.

APPENDIX D

Conversion of International volumes and growth to Spaulding log rule

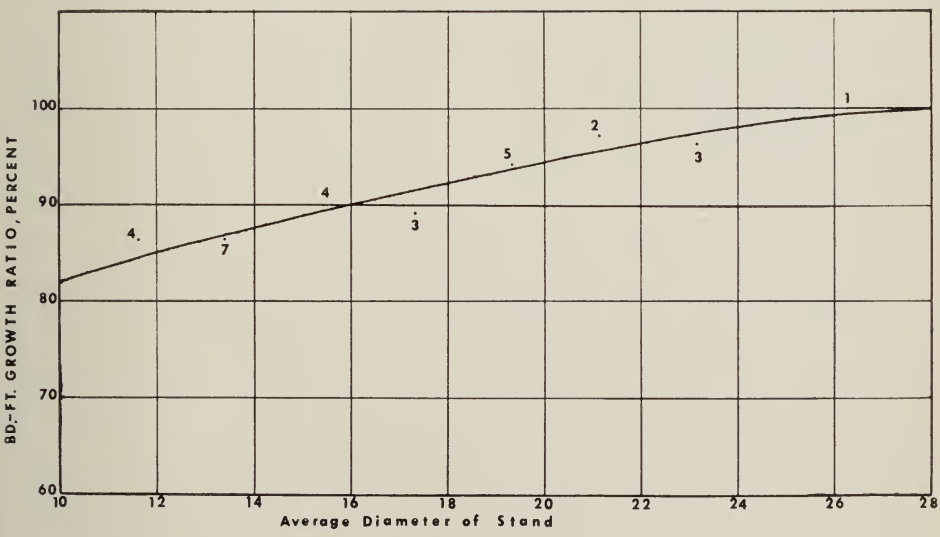
Comparisons of individual tree volumes for a given total height and diameter of the International ¼″ Rule (table 27) and the Spaulding Rule



7. Curves for conversion of International ¼" Rule volumes to Spaulding rule, volumes for trees and stands over diameter.

(table 26), indicate, over the ranges of heights and diameters, that the International Rule stem volume is consistently greater. Expressing Spaulding volumes as a percentage of the International reveals that the Spaulding volumes are nearly 100 per cent by the time the trees reach 50 inches DBH. This change in tree volume percentage is shown graphically in figure 7, line A. Expanding the individual trees into a stand structure with associated ranges of diameters brings about a change in the percentage relationships; these are shown in figure 7, line B. The line expressing the percentage relationship of stands was developed from average diameters and volume ratios of 27 remeasured sample plots whose inventory volumes were computed using both log rules. The average diameter refers to the stand larger than 10.5 inches DBH.

Predictions of board-foot volume growth in this study are made on the basis of the volume tables developed



8. Curve of Spaulding volume growth to International ¼" growth over average diameter of stands.

from the International log rule. Conversion of stand volume growth to the Spaulding rule may be necessary for those who prefer to express predictions in terms of this more commonly used log rule of the Redwood Region. Basis for accomplishing this conversion is a curve of the percentage volume ratios of stand growth over the average stand diameter of trees larger than 4.5 inches DBH (figure 8). This curve has been developed from obsrevations of permanent remeasured growth plots whose net volume growth rates were developed using volume tables of the two log rules considered. Application of this conversion curve requires an estimate of the average stand diameter; this may be secured from stand measurements or from table 4 of the young-growth redwood yield tables (Lindquist and Palley, 1963). The volume growth ratio, as a percentage, is read from the curve and multiplied by the predicted International rule volume growth to give the stand growth in terms of the Spaulding rule. It is apparent that once stands have reached nearly 28 inches average diameter (trees larger than 4.5 inches DBH) that the volume growth for the two rules are identical; average stand diameter less than this amount will require conversion. For example, predicted net board-foot volume growth of a stand 20 inches in diameter at 60 years, site index 160, and initial basal area of 400 square feet is equal to 25,759 board-feet for the 10-year period. The volume ratio for an average diameter of 20 inches is approximately 94 per cent; Spaulding net volume growth for the period is: 25,759 × 94 per cent = 24,213 board-feet.

APPENDIX E

Volume tables

Stem volume computations for deriving the local volume lines for each sample have been carried out as an internal function of the basic computing program. In accomplishing this the regression coefficients expressing appropriate standard volume tables (based on total height and diameter) have been made available to the program. Published volume tables for young-growth redwood (Palley, 1959, 1961), tables 25, 26, and 27, were developed by multiple regression analysis; these equations were available for the cubic- and board-foot volume computations

Table 24. Summary of the Stem Volume Regression Coefficients of Conifers, and Local Volume Line Coefficients of Hardwoods. All Board-Foot Volumes Based on the International 1/4 Inch Rule

Species	Unit	Regression coefficients					
		Constant	D	H	D.H.	D^2	$D^2 \cdot H$
Redwood	cu ft	−1.3803	.26709	.00167	.001956	−.016521	.00145
Redwood	bd ft	.....	7.09140	−1.96810	.084360	−.576750	.011580
Whitewood	cu ft	−6.7903	1.37294	.....	.....	−.05245	.00194
Whitewood	bd ft	−248.289	55.0921	−1.01236	.....	−3.00570	.02467
Hardwood	cu ft	−2.16	.....	.....	.....	.1354	.....
Hardwood	bd ft	−69.8	.....	.....	.....	.698	.....

D = diameter at breast height (inches)
H = total height of dominants (feet)

Table 25. MERCHANTABLE CUBIC-FOOT VOLUME TABLE FOR YOUNG-GROWTH REDWOOD
(*Sequoia sempervirens* [D. Don] Endl.)
Cubic foot volume to a 4-inch top inside bark

D.B.H. (inches)	Total height (feet)																		No. of trees
	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	
4		.3	.7	1.0	1.3														2
5	.4	.9	1.4	1.8	2.3														6
6	.9	1.6	2.2	2.9	3.6	4.2													2
7		2.3	3.1	4.0	4.9	5.7													4
8		3.0	4.1	5.2	6.3	7.4	8.5												9
9		3.9	5.2	6.6	7.9	9.3	11												6
10		4.7	6.4	8.1	9.8	11	13												4
11		5.6	7.6	9.6	12	14	16	18	20	22									9
12		6.6	8.9	11	14	16	18	21	23	25									9
13		7.7	10	13	16	18	21	24	27	29	32	35							8
14		8.7	12	15	18	21	24	28	31	34	37	40							16
15		10	13	17	21	24	28	31	35	38	42	46							13
16		11	15	19	23	27	31	35	39	43	48	52	56						20
17		12	17	22	26	30	35	40	44	49	53	58	62						16
18		14	19	24	29	34	39	44	49	54	59	64	69	74					20
19		15	21	26	32	38	43	49	54	60	66	71	77	82					21
20		16	23	29	35	42	47	54	60	66	72	78	85	91					22
21		18	25	32	38	45	52	59	66	72	79	86	93	100					14
22		19	27	34	42	49	57	64	72	79	87	94	102	109					19
23		21	29	37	45	54	62	70	78	86	94	102	110	119	127	135			14
24		23	32	40	49	58	67	76	84	93	102	111	120	129	138	146			31
25		24	34	43	53	62	72	82	91	101	110	120	130	139	149	158			21
26		26	36	47	57	67	78	88	98	109	119	129	140	150	160	171	181		27
27			39	50	61	72	83	94	106	117	128	139	150	161	172	183	194		25
28			41	53	65	77	89	101	113	125	137	149	161	173	185	197	208		36
29			44	57	70	82	95	108	121	134	146	159	172	185	198	210	223		18
30			47	61	74	88	102	115	129	143	156	170	184	197	211	224	238	252	24
31				64	79	94	108	123	137	152	166	181	196	210	225	239	254	268	17
32				68	84	99	115	130	146	161	177	192	208	223	239	254	270	285	32
33				72	89	105	122	138	154	171	188	204	220	237	253	270	286	303	16
34				76	94	111	129	146	164	181	199	216	233	251	268	286	303	321	24
35				81	99	118	136	154	173	191	210	228	247	265	284	302	321	339	16
36				85	104	124	144	163	182	202	222	241	261	280	300	319	339	358	13
37					110	130	151	172	192	213	234	254	275	295	316	336	357	378	8
38					116	137	159	181	202	224	246	267	289	311	332	354	376	398	14
39					121	144	167	190	213	236	258	281	304	327	350	373	395	418	1
40					127	151	175	199	223	247	271	295	319	343	367	391	415	439	4
41					133	158	184	209	234	259	284	310	335	360	385	410	436	461	2
42					139	166	192	219	245	271	298	324	351	377	404	430	456	483	4
43						173	201	229	256	284	312	339	367	394	422	450	478	505	3
44						181	210	239	268	297	326	355	384	412	442	470	499	528	4
45						189	219	249	280	310	340	370	400	431	461	491	522	552	4
46						197	228	260	292	323	355	386	418	450	481	513	544	576	4
47						205	238	271	304	337	370	403	436	468	502	534	568	600	1
48						213	247	282	316	350	385	419	454	488	522	557	591	626	2
49						221	257	293	329	365	400	436	472	508	544	579	615	651	1
50						230	267	305	342	379	416	454	491	528	565	603	640	677	7
No. of trees		5	10	13	19	41	32	20	69	90	122	94	45	13	13	6	1		593

Table entries may be approximated by the equation:
$V = -1.380326 + 0.267097D + 0.001675H + 0.001956DH - 0.016521D^2 + 0.0014505D^2H$, where V is the volume of solid wood (in cubic feet) between stump and top, D is d.b.h. (in inches) on the uphill side, and H is height (in feet) from ground level on uphill side to tip.

Log rule: bott logs cubed by Newton's formula, upper logs by Smalian's.
Scaling length: mostly 16 feet.
Trim allowance: none.
Stump height: 1.5 feet, uphill side.
Top diameter: 4.0 inches, inside bark.
Basis: 593 trees.
Data gathered in Sonoma, Mendocino, Humboldt, and Del Norte counties, California.

Table 26. Total Height Volume Table for Young Growth Redwood
(*Sequoia sempervirens* [D. Don] Endl.)
Board feet to an 8-inch top—Spaulding Rule

D.B.H. in inches	Volume by total height in feet																No. of trees
	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	
11	20	22	25	27	29	32	34	36									9
12	25	31	36	41	46	51	56	61									9
13	31	39	47	55	64	72	80	88	96								8
14	36	49	59	71	82	94	105	117	128								16
15	42	57	72	87	102	117	132	147	162	177							13
16	47	66	85	104	123	142	160	179	198	217							20
17	52	75	98	121	144	167	190	213	236	259	283						16
18	57	85	112	139	167	194	222	249	277	304	331						20
19	62	94	126	159	191	223	255	287	319	351	383						21
20	67	104	141	178	215	252	289	326	363	400	437						22
21		115	157	199	241	283	325	367	409	452	494	536					14
22		125	172	220	268	315	363	410	458	505	553	601					19
23		136	189	242	295	349	402	455	508	562	615	668					14
24		146	206	265	324	383	442	502	561	620	679	739					31
25		157	223	288	354	419	485	550	616	681	746	812	877				21
26			241	313	384	456	528	600	672	744	816	888	960				27
27			259	337	416	495	574	652	731	810	888	967	1,046				25
28			277	363	449	535	620	706	792	877	963	1,049	1,134				36
29			297	390	483	576	669	762	855	948	1,040	1,133	1,226	1,319			18
30			316	417	517	618	718	819	920	1,020	1,121	1,221	1,322	1,422			24
31				445	553	661	770	878	986	1,095	1,203	1,311	1,420	1,528	1,636	1,745	17
32				474	590	706	823	939	1,056	1,172	1,288	1,405	1,521	1,637	1,754	1,870	32
33				503	628	752	877	1,002	1,127	1,251	1,376	1,501	1,626	1,750	1,875	2,000	16
34				533	666	800	933	1,066	1,200	1,333	1,466	1,600	1,733	1,866	2,000	2,133	24
35				564	706	848	991	1,133	1,275	1,417	1,559	1,702	1,844	1,986	2,128	2,270	16
36					747	898	1,050	1,201	1,352	1,504	1,655	1,806	1,958	2,109	2,260	2,412	13
37					789	949	1,110	1,271	1,432	1,592	1,753	1,914	2,075	2,235	2,396	2,557	8
38					832	1,002	1,172	1,343	1,513	1,683	1,854	2,024	2,195	2,365	2,535	2,706	14
39					875	1,056	1,236	1,416	1,597	1,777	1,957	2,137	2,318	2,498	2,678	2,859	1
40					920	1,111	1,301	1,492	1,682	1,873	2,063	2,254	2,444	2,634	2,825	3,015	4
41						1,167	1,368	1,569	1,770	1,971	2,172	2,373	2,573	2,774	2,975	3,176	2
42						1,224	1,436	1,648	1,859	2,071	2,283	2,494	2,706	2,917	3,129	3,341	4
43						1,282	1,505	1,728	1,940	2,173	2,396	2,618	2,841	3,063	3,286	3,509	3
44						1,343	1,577	1,811	2,045	2,279	2,513	2,746	2,980	3,214	3,448	3,682	4
45						1,405	1,650	1,895	2,141	2,386	2,631	2,877	3,122	3,367	3,613	3,858	4
46							1,724	1,982	2,239	2,496	2,753	3,010	3,267	3,524	3,781	4,039	4
47							1,800	2,070	2,339	2,608	2,877	3,146	3,415	3,684	3,954	4,223	1
48							1,878	2,159	2,441	2,722	3,004	3,285	3,567	3,848	4,129	4,411	2
49							1,957	2,251	2,545	2,839	3,133	3,427	3,721	4,015	4,309	4,603	1
50							2,037	2,344	2,651	2,958	3,265	3,572	3,878	4,185	4,492	4,799	7
No. of trees	2	4	10	39	32	20	69	90	122	94	45	13	13	6	1		560

Table prepared by weighted multiple regression. Table entries computed from the equation:
$V = 7.0015D - 1.0930H - 0.02451DH - 0.55260D^2 + 0.01320D^2H$, where D is d.b.h. on the uphill side and H is height in feet from ground level on uphill side to tip.
Log rule—Spaulding. (Scribner values used for 8- and 9-inch logs.)
Scaling length—16 feet.
Trim allowance—0.3 feet.
Stump height—1.5 feet, uphill side.
Top diameter—8.0 inches, inside bark.
Basis—560 trees.
Data gathered in Sonoma, Mendocino, Humboldt, and Del Norte counties, California.
Aggregate difference—Table is 0.11 per cent high.
Average deviation—15.59 per cent.
Heavy line shows range of basic data.

Table 27. Total Height Volume Table for Young Growth Redwood
(*Sequoia sempervirens* [D. Don] Endl.)
Board feet to an 8-inch top—International 1/4 Inch Rule

D.B.H. in inches	Volume by total height in feet																No. of trees
	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	
11	25	25	30	35	40	40	45	50									9
12	30	40	45	50	60	65	75	80									9
13	40	50	60	70	80	95	105	115	125								8
14	45	60	75	90	105	120	135	150	165								16
15	55	70	90	110	130	150	165	185	205	225							13
16	60	85	105	130	155	175	200	225	250	270							20
17	65	95	125	150	180	210	235	265	290	320	350						16
18	75	105	140	170	205	240	270	305	340	370	405						20
19	80	120	155	195	230	270	310	345	385	425	460						21
20	85	130	175	215	260	305	345	390	435	475	520						22
21		140	190	240	290	335	385	435	485	535	585	630					14
22		150	205	260	315	370	425	480	535	590	645	700					19
23		165	225	285	345	405	470	530	590	650	710	775					14
24		175	240	310	375	445	510	580	645	715	780	850					31
25		185	260	335	410	480	555	630	705	775	850	925	1,000				21
26			280	360	440	520	600	680	760	840	925	1,005	1,085				27
27			295	385	470	560	645	735	820	910	995	1,085	1,170				25
28			315	410	505	600	695	790	885	980	1,075	1,170	1,265				36
29			335	435	540	640	745	845	945	1,050	1,150	1,255	1,355	1,460			18
30			355	465	575	685	795	905	1,010	1,120	1,230	1,340	1,450	1,560			24
31				490	610	725	845	960	1,080	1,195	1,315	1,430	1,550	1,670	1,785	1,905	17
32				520	645	770	895	1,020	1,150	1,275	1,400	1,525	1,650	1,775	1,905	2,030	32
33				545	680	815	950	1,085	1,220	1,350	1,485	1,620	1,755	1,890	2,025	2,160	16
34				575	720	860	1,005	1,145	1,290	1,430	1,575	1,720	1,860	2,005	2,145	2,290	24
35				605	755	905	1,060	1,210	1,365	1,515	1,665	1,820	1,970	2,120	2,275	2,425	16
36					795	955	1,115	1,275	1,440	1,600	1,760	1,920	2,080	2,240	2,400	2,565	13
37					835	1,005	1,175	1,345	1,515	1,685	1,855	2,025	2,195	2,365	2,535	2,705	8
38					875	1,055	1,235	1,415	1,590	1,770	1,950	2,130	2,310	2,490	2,670	2,850	14
39					915	1,105	1,295	1,485	1,670	1,860	2,050	2,240	2,430	2,620	2,810	3,000	1
40					955	1,155	1,355	1,555	1,755	1,955	2,150	2,350	2,550	2,750	2,950	3,150	4
41						1,210	1,415	1,625	1,835	2,045	2,255	2,465	2,675	2,885	3,095	3,305	2
42						1,260	1,480	1,700	1,920	2,140	2,360	2,580	2,800	3,020	3,240	3,460	4
43						1,315	1,545	1,775	2,005	2,240	2,470	2,700	2,930	3,160	3,390	3,620	3
44						1,370	1,610	1,855	2,095	2,335	2,580	2,820	3,060	3,305	3,545	3,785	4
45						1,425	1,680	1,930	2,185	2,440	2,690	2,945	3,195	3,450	3,700	3,955	4
46							1,750	2,010	2,275	2,540	2,805	3,070	3,335	3,595	3,860	4,125	4
47							1,820	2,095	2,370	2,645	2,920	3,195	3,470	3,750	4,025	4,300	1
48							1,890	2,175	2,465	2,750	3,040	3,325	3,615	3,900	4,190	4,475	2
49							1,960	2,260	2,560	2,860	3,160	3,460	3,760	4,060	4,360	4,660	1
50							2,035	2,345	2,655	2,970	3,280	3,595	3,905	4,215	4,530	4,840	7
No. of trees	2	4	10	39	32	20	69	90	122	94	45	13	13	6	1		560

Table prepared by weighted multiple regression. Table entries computed from the equation:
$V = 7.0914D - 1.9681H + 0.08436DH - 0.57675D^2 + 0.01158D^2H$, where D is d.b.h. on the uphill side and H is height in feet from ground level on uphill side to tip.
Log rule—International ¼-inch.
Scaling length—16 feet.
Trim allowance—0.3 feet.
Stump height—1.5 feet, uphill side.
Top diameter—8.0 inches, inside bark.
Basis—560 trees.
Data gathered in Sonoma, Mendocino, Humboldt, and Del Norte counties.
Aggregate Difference—Table is 0.06 per cent low.
Average Deviation—14.72 per cent.
Heavy line shows range of basic data.

carried out in this study. Stem volumes for representation of all other conifers used those of Douglas fir presented in Agricultural Handbook No. 92, table 5, for cubic-foot volumes, and table 9 for International ¼″ volumes. Expression of the Douglas fir tables in an equation form was done by developing multiple regression equations based on measured total heights and diameters of several stems of whitewood included in the sampling of these stands considered. A summary of the stem volume equations, as calculated and used in this project, is shown in table 24 entries 1–4. Also shown in this table, entries 5 and 6, are the local volume line coefficients used for the computation of stand hardwood volumes.[5] These hardwood volume line coefficients were inserted in equation 8 for computation of the hardwood segment of the total stand volumes. Since total height was not measured on sampled hardwoods, it was not possible to follow the procedure used for computation of conifer volumes; these hardwood coefficients were used as constants for all samples and inventories in which hardwood stems appeared.

APPENDIX F

Prediction error estimation

The use of the growth prediction equations shown in tables 3, 6, 9, and 12 to derive estimates of the net 10 year stand growth does not provide an indication of the degree of variation in the growth estimates resulting from the errors of the regression equations and the sampling error of the user's set of stand samples. To accomplish this estimation of the variability of the predictions requires the elements of the inverse of the matrix of $(X_i - \bar{X}_i) \times (X_j - \bar{X}_j)$ terms of the original set of observations from which the predictions equations were derived. This matrix is often referred to as the matrix of Gauss multipliers (Snedecor, 1956), and the elements of this matrix noted as C_{ij}. These elements have been computed from the sample data and are presented in the following tables 28, 29, 30, and 31. Also shown in these tables are the mean values of the independent variables of the N set of samples $(\bar{X}_i)$.

Table 28. Multipliers for Logarithm of Basal Area Growth

Variable	Log BA	Log age	$\bar{X}$
Log basal area....	.149046	−.154439	2.3924
Log age.....	−.154439	.270168	1.5454

The multipliers may be computed by a procedure referred to as the "Doolittle method" of inverting a matrix. However, in this study an alternative procedure was utilized. The procedure used statistics readily available from the multiple regression analysis program used in this study. The inverse matrix of the simple correlation coefficient matrix, (r_{ij}), is printed and noted as $(r_{ij})^{-1}$ where:

$$(r_{ij})^{-1} \cdot (r_{ij}) = (I) \tag{15}$$

and

$$r_{ij} = \frac{\Sigma(X_i - \bar{X}_i) \cdot (X_j - \bar{X}_j)}{\sqrt{\Sigma(X_i - \bar{X}_i)^2 \cdot (X_j - \bar{X}_j)^2}}$$

The elements of the covariance matrix

[5] Derived from local volume tables of tanoak (*Lithocarpus densiflorus*) and red alder (*Alnus rubra*) on file with the Pacific Southwest Forest and Range Experiment Station.

Table 29. Multipliers for Log of Cubic-Foot Volume Growth

Variable	Log BA	Log site	Log age	$\bar{X}$
Log basal area	.210785	−.388372	−.212809	2.3924
Log of site	−.388372	2.44336	.367179	2.2394
Log of age	−.212809	.367179	.325353	1.5454

Table 30. Multipliers for Board-Foot Growth of Trees Larger than 10.5 Inches DBH
(Values scaled by 10^{-10}, except $\bar{X}$)

Variable	BA²	Site²	BA × Age	Age × Site	BA × Site	$\bar{X}$
BA × BA	.09227	.13996	−.71362	1.9607	.26288	131,000
Site × Site	.13996	2.0218	−.38197	−1.7339	−.82177	31,200
BA × Age	−.71362	−.38197	6.8852	−18.797	−1.6871	15,800
Age × Site	1.9607	−1.7339	−18.797	62.933	−5.4852	7,170
BA × Site	.26288	−.82177	−1.6871	−5.4852	.99475	55,800

Table 31. Multipliers for Board-Foot Growth of Trees Larger than 15.5 Inches DBH
(Values scaled by 10^{-10}, except $\bar{X}$)

Variable	BA²	Site²	BA × Age	Age × Site	BA × Site	$\bar{X}$
BA × BA	.09313	.1427	−.7213	1.989	−.2659	134,000
Site × Site	.1427	2.115	−.3151	1.508	−.8649	31,300
BA × Age	−.7210	−.3151	7.069	−1.994	1.671	16,200
Age × Site	1.989	1.508	−1.994	65.39	−5.436	7,310
BA × Site	−.2659	−.8649	1.671	−5.436	1.019	56,900

of the independent variables are evaluated by:

$$S^b_{ij} = \frac{S^2_{y\cdot x}}{N \cdot SD_i \cdot SD_j} \cdot (r_{ij})^{-1} \qquad (16)$$

where:

SD_i = the standard deviation of variable X_i

N = the number of sample observations

then, since

$$C_{ij} = S^b_{ij}/S^2_{y\cdot x}$$

we can substitute to give the relationship of

$$C_{ij} = \frac{(r_{ij})^{-1}}{N \cdot SD_i \cdot SD_j} \qquad (17)$$

Thus the only values needed for the solution of the matrix of multipliers is the inverse of the correlation coefficient matrix and standard deviations of the independent variables; both are printed by the regression analysis program.

Tables 28–31 show values of the C_{ij}'s and $\bar{X}_i$'s needed for the solution of the procedure for deriving the estimate of the variance of the predicted growth.

The operational equation for the derivation of the variance of the predicted growth for two independent variables is shown in the text equation (5). Similar equations suitable for the three and five independent variables of the prediction equations follow the same basic pattern. These variance

estimators are derived from the following matrix notation:

$$S_{\hat{y}}^2 = SE^2_{\text{est}}/N + \frac{1}{M}\left[(b_1\, b_2 \ldots b_p) \cdot \begin{bmatrix} S_{11}\, S_{12} \ldots S_{1p} \\ S_{12}\, S_{22} \ldots S_{2p} \\ \ldots\ldots\ldots\ldots \\ S_{1p}\, S_{2p} \ldots S_{pp} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ \ldots \\ b_p \end{bmatrix} \right] \tag{18}$$

$$+ SE^2_{\text{est}} \left[[(X_1 - \bar{X}_1)(X_2 - \bar{X}_2) \ldots (X_p - \bar{X}_p)] \cdot \begin{bmatrix} C_{11}\, C_{12} \ldots C_{1p} \\ C_{12}\, C_{22} \ldots C_{2p} \\ \ldots\ldots\ldots\ldots \\ C_{1p}\, C_{2p} \ldots C_{pp} \end{bmatrix} \cdot \begin{bmatrix} (X_1 - \bar{X}_1) \\ (X_2 - \bar{X}_2) \\ \ldots\ldots\ldots \\ (X_p - \bar{X}_p) \end{bmatrix} \right]$$

$$- SE^2_{\text{est}}/M \left[[(1\ 1 \ldots 1)] \cdot \begin{bmatrix} S_{11}\, S_{12} \ldots S_{1p} \\ S_{12}\, S_{22} \ldots S_{2p} \\ \ldots\ldots\ldots\ldots \\ S_{1p}\, S_{2p} \ldots S_{pp} \end{bmatrix} \cdot \begin{bmatrix} C_{11}\, C_{12} \ldots C_{1p} \\ C_{12}\, C_{22} \ldots C_{2p} \\ \ldots\ldots\ldots\ldots \\ C_{1p}\, C_{2p} \ldots C_{pp} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 1 \\ \cdot \\ 1 \end{bmatrix} \right]$$

where:

$S_{\hat{y}}^2$ = variance of predicted mean growth of M set samples
SE^2_{est} = standard error of estimate of the N set
S_{1p} = variance and covariance matrix of M set variables
C_{1p} = matrix elements of multipliers derived from N set.